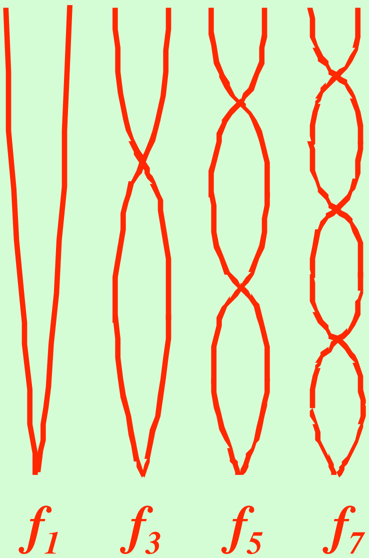
Start Here

THE PHYSICS OF MUSIC AND MUSICAL INSTRUMENTS



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"Everything is determined … by forces over which we have no control. It is determined for the insects as well as for the star. Human beings, vegetables, or cosmic dust – we all dance to a mysterious tune, intoned in the distance by an invisible piper."
– Albert Einstein

INTRODUCTION

THIS MANUAL COVERS the physics of waves, sound, music, and musical instruments at a level designed for high school physics. However, it is also a resource for those teaching and learning waves and sound from middle school through college, at a mathematical or conceptual level. The mathematics required for full access to the material is algebra (to include logarithms), although each concept presented has a full conceptual foundation that will be useful to those with even a very weak background in math.

Solomon proclaimed that there is nothing new under the Sun and of the writing of books there is no end. Conscious of this, I have tried to produce something that is not simply a rehash of what has already been done elsewhere. In the list of references I have indicated a number of very good sources, some classics that all other writers of musical acoustic books refer to and some newer and more accessible works. From these, I have synthesized what I believe to be the most useful and appropriate material for the high school aged student who has neither a background in waves nor in music, but who desires a firm foundation in both. Most books written on the topic of musical acoustics tend to be either very theoretical or very cookbook style. The theoretical ones provide for little student interaction other than some end of the chapter questions and problems. The ones I term "cookbook" style provide instructions for building musical instruments with little or no explanation of the physics behind the construction. This curriculum attempts to not only marry the best ideas from both types of books, but to include pedagogical aids not found in other books.

This manual is available as both a paper hard copy as well as an e-book on CD-ROM. The CD-ROM version contains hyperlinks to interesting websites related to music and musical instruments. It also contains hyperlinks throughout the text to sound files that demonstrate many concepts being developed.

MODES OF PRESENTATION

As the student reads through the text, he or she will encounter a number of different presentation modes. Some are color-coded. The following is a key to the colors used throughout the text:

Pale green boxes cover tables and figures that are important reference material.

Notes	Frequency interval (cents)
C_i	0
D	204
E	408
F	498
G	702
A	906
B	1110
C_f	1200

Table 2.8: Pythagorean scale interval ratios

Light yellow boxes highlight derived equations in their final form, which will be used for future calculations.

$$f_1 = \frac{\sqrt{\frac{T}{\mu}}}{2L}$$

Tan boxes show step-by-step examples for making calculations or reasoning through questions.

Example

If the sound intensity of a screaming baby were $1\times10^{-2}\ \frac{W}{m^2}$ at 2.5 m away, what would it be at 6.0 m away?

The distance from the source of sound is greater by a factor of $\frac{6.0}{2.5} = 2.4$. So the sound intensity is decreased by $\frac{1}{(2.4)^2} = 0.174$. The new sound intensity is:

$$(1\times10^{-2}\ \frac{W}{m^2})(0.174) = \boxed{1.74\times10^{-2}\ \frac{W}{m^2}}$$

Gray boxes throughout the text indicate stopping places in the reading where students are asked, “Do you get it?” The boxes are meant to reinforce student understanding with basic recall questions about the immediately preceding text. These can be used to begin a discussion of the reading with a class of students.

Do you get it? (4)

A solo trumpet in an orchestra produces a sound intensity level of 84 dB. Fifteen more trumpets join the first. How many decibels are produced?

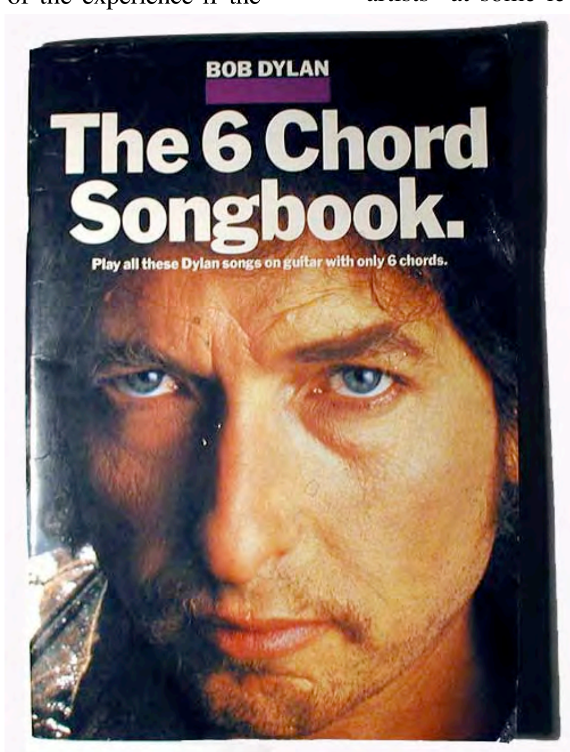
In addition to the **“Do you get it?”** boxes, which are meant to be fairly easy questions done individually by students as they read through the text, there are three additional interactions students will encounter: **Activities**, **Investigations**, and **Projects**. Activities more difficult than the “Do you get it?” boxes and are designed to be done either individually or with a partner. They either require a higher level of conceptual understanding or draw on more than one idea. Investigations are harder still and draw on more than an entire section within the text. Designed for two or more students, each one photographically exposes the students to a particular musical instrument that they must thoroughly consider. Investigations are labs really, often requiring students to make measurements directly on the photographs. Solutions to the “Do you get it?” boxes, Activities, and Investigations are provided in an appendix on the CD-ROM. Finally, projects provide students with some background for building musical instruments, but they leave the type of musical scale to be used as well as the key the instrument will be based on largely up to the student.

PHYSICS AND … MUSIC?

“Without music life would be a mistake.”
– Friedrich Nietzsche

With even a quick look around most school campuses, it is easy to see that students enjoy music. Ears are sometimes hard to find, covered by headphones connected to radios or portable CD players. And the music flowing from them has the power to inspire, to entertain, and to even mentally transport the listener to a different place. A closer look reveals that much of the life of a student either revolves around or is at least strongly influenced by music. The radio is the first thing to go on in the morning and the last to go off at night (if it goes off at all). T-shirts with logos and tour schedules of popular bands are the artifacts of many teens' most coveted event … the concert. But the school bell ringing for the first class of the day always brings with it a stiff dose of reality.

H. L. Mencken writes, "School days, I believe, are the unhappiest in the whole span of human existence. They are full of dull, unintelligible tasks, new and unpleasant ordinances, brutal violations of common sense and common decency." This may paint too bleak a picture of the typical student's experience, but it's a reminder that what is taught often lacks meaning and relevance. When I think back to my own high school experience in science, I find that there are some classes for which I have no memory. I'm a bit shocked, but I realize that it would be possible to spend 180 hours in a science classroom and have little or no memory of the experience if the classroom experience were lifeless or disconnected from the reality of my life. Middle school and high school students are a tough audience. They want to be entertained … but they don't have to be. What they really need is relevance. They want to see direct connections and immediate applications. This is the reason for organizing an introduction to the physics of waves and sound around the theme of music and musical instruments.



It's not a stretch either. Both music and musical instruments are intimately connected to the physics of waves and sound. To fully appreciate what occurs in a musical instrument when it makes music or to understand the rationale for the development of the musical scales one needs a broad foundation in most elements of wave and sound theory. With that said, the approach here will be to understand music and musical instruments first, and to study the physics of waves and sound as needed to push the understanding of the music concepts. The goal however is a deeper understanding of the physics of waves and sound than what would be achieved with a more traditional approach.

SOUND, MUSIC, AND NOISE

Do you like music? No, I guess a better question is, what kind of music do you like? I don't think anyone dislikes music. However, some parents consider their children's "music" to be just noise. Likewise, if the kids had only their parent's music to listen to many would avoid it in the same way they avoid noise. Perhaps that's a good place to start then – the contrast between music and noise. Is there an objective, physical difference between music and noise, or is it simply an arbitrary judgment?

After I saw the movie *8 Mile*, the semi-autobiographical story of the famous rapper Eminem, I recommended it to many people … but not to my mother. She would have hated it. To her, his music is just noise. However, if she hears an old Buddy Holly song, her toes start tapping and she's ready to dance. But the music of both of these men would be considered unpleasant by my late grandmother who seemed to live for the music she heard on the Lawrence Welk Show. I can appreciate all three "artists" at some level, but if you ask me, nothing beats a little Bob Dylan. It's obviously not easy to define the difference between noise and music. Certainly there is the presence of rhythm in the sounds we call music. At a more sophisticated level there is the presence of tones that combine with other tones in an orderly and ... "pleasing" way. Noise is often associated with very loud and grating sounds – chaotic sounds which don't sound good together or are somehow "unpleasant" to listen to. Most would agree that the jackhammer tearing up a portion of the street is noise and the sound coming from the local marching band is music. But the distinction is much more subtle than that. If music consists of sounds with rhythmic tones of certain frequencies then the jackhammer might be considered a musical instrument. After all, it pummels the street with a very regular frequency. And if noise consists of loud sounds, which are unpleasant to listen to, then the cymbals used to punctuate the performance of the marching band might be considered noise. I suppose we all define the point where music becomes noise a bit differently. Perhaps it's based on what we listen to most or on the generation we grow up in or … make a break from. But we need to be careful about being cavalier as I was just now when I talked about "pleasing" sounds. John Bertles of Bash the Trash® (a company dedicated to the construction and performance of musical instruments from recycled materials: http://www.bashthetrash.com/) was quick to caution me when I used the word "pleasing" to describe musical sound. Music that is pleasing to one person may not be pleasing to others. Bertles uses the definition of *intent* rather than *pleasing* when discussing musical sound. He gives the example of a number of cars all blaring their horns chaotically at an intersection. The sound would be considered noise to most anyone. But the reason for the noise is not so much the origin of the sound, but the lack of intent to organize the sounds of the horns. If someone at the intersection were to direct the car horns to beep at particular times and for specific periods, the noise would evolve into something more closely related to music. And no one can dispute that whether it's Eminem, Buddy Holly, Lawrence Welk, or Bob Dylan, they all create(d) their particular recorded sounds with intent.

"There are two means of refuge from the miseries of life: music and cats."
– Albert Schweitzer

BEGINNING TO DEFINE MUSIC

Music makes us feel good, it whisks us back in time to incidents and people from our lives; it rescues us from monotony and stress. Its tempo and pace jive with the natural rhythm of our psyche.

The simplest musical sound is some type of rhythmical beating. The enormous popularity of the stage show *Stomp* http://www.stomponline.com/ and the large screen Omnimax movie, *Pulse* http://www.Pulsethemovie.com/ gives evidence for the vast appreciation of this type of music. Defining the very earliest music and still prominent in many cultures, this musical sound stresses beat over melody, and may in fact include no melody at all. One of the reasons for the popularity of rhythm-only music is that anyone can immediately play it at some level, even with no training. Kids do it all the time, naturally. The fact that I often catch myself spontaneously tapping my foot to an unknown beat or lie in bed just a bit longer listening contentedly to my heartbeat is a testament to the close connection between life and rhythm.

Another aspect of music is associated with more or less pure tones – sounds with a constant pitch. Whistle very gently and it sounds like a flute playing a single note. But that single note is hardly a song, and certainly not a melody or harmony. No, to make the single tone of your whistle into a musical sound you would have to vary it in some way. So you could change the way you hold your mouth and whistle again, this time at a different pitch. Going back and forth between these two tones would produce a cadence that others might consider musical. You could whistle one pitch in one second pulses for three seconds and follow that with a one second pulse of the other pitch. Repeating this pattern over and over would make your tune more interesting. And you could add more pitches for even more sophistication. You could even have a friend whistle with you, but with a different pitch that sounded good with the one you were whistling.

If you wanted to move beyond whistling to making music with other physical systems, you could bang on a length of wood or pluck a taut fiber or blow across an open bamboo tube. Adding more pieces with different lengths (and tensions, in the case of the fiber) would give additional tones. To add more complexity you could play your instrument along with other musicians. It might be nice to have the sound of your instrument combine nicely with the sound of other instruments and have the ability to play the tunes that others come up with. But to do this, you would have to agree on a collection of common pitches. There exist several combinations of common pitches. These are the **musical scales**.

Here we have to stop and describe what the pitch of a sound is and also discuss the various characteristics of sound. Since sound is a type of wave, it's additionally necessary to go even further back and introduce the idea of a wave.

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"It is only by introducing the young to great literature, drama and music, and to the excitement of great science that we open to them the possibilities that lie within the human spirit – enable them to see visions and dream dreams."
– Eric Anderson

CHAPTER 1
WAVES AND SOUND

ASK MOST PEOPLE to define what a wave is and they get a mental image of a wave at the beach. And if you really press them for an answer, they're at a loss. They can describe what a wave looks like, but they can't tell you what it is. What do you think?

Figure 1.1: Most people think of the ocean when asked to define or describe a wave. The recurring tumult is memorable to anyone who has spent time at the beach or been out in the surf. But waves occur most places and in many different forms, transferring energy without transferring matter.

If you want to get energy from one place to another you can do it by transferring it with some chunk of matter. For example, if you want to break a piece of glass, you don't have to physically make contact with it yourself. You could throw a rock and the energy you put into the rock would travel on the rock until it gets to the glass. Or if a police officer wants to subdue a criminal, he doesn't have to go up and hit him. He can send the energy to the criminal via a bullet. But there's another way to transfer energy – and it doesn't involve a transfer of matter. The other way to transfer energy is by using a wave. *A wave is a transfer of energy without a transfer of matter*. If you and a friend hold onto both ends of a rope you can get energy down to her simply by moving the rope back and forth. Although the rope has some motion, it isn't actually transferred to her, only the energy is transferred.

A tsunami (tidal wave generally caused by an earthquake) hit Papua New Guinea in the summer of 1998. A magnitude 7 earthquake 12 miles offshore sent energy in this 23-foot tsunami that killed thousands of people. Most people don't realize that the energy in a wave is proportional to the square of the amplitude (height) of the wave. That means that if you compare the energy of a 4-foot wave that you might surf on to the tsunami, even though the tsunami is only about 6 times the height, it would have 6^2 or 36 times more energy. A 100-foot tsunami (like the one that hit the coast of the East Indies in August 1883) would have 25^2 or 625 times more energy than the four-foot wave.

Streaming through the place you're in right now is a multitude of waves known as electromagnetic waves. Their wavelengths vary from so small that millions would fit into a millimeter, to miles long. They're all here, but you miss most of them. The only ones you're sensitive to are a small group that stimulates the retinas of your eyes (visible light) and a small group that you detect as heat (infrared). The others are totally undetectable. But they're there.

WAVES, SOUND, AND THE EAR

Another type of wave is a sound wave. As small in energy as the tsunami is large, we usually need an ear to detect these. Our ears are incredibly awesome receptors for sound waves. The threshold of hearing is somewhere around $1 \times 10^{-12} Watts/meter^2$. To understand this, consider a very dim 1-watt night-light. Now imagine that there were a whole lot more people on the planet than there are now – about 100 times more. Assume there was a global population of 1 trillion (that's a million, million) people. If you split the light from that dim bulb equally between all those people, each would hold a radiant power of 1×10^{-12} *Watts*. Finally, let's say that one of those people spread that power over an area of one square meter. At this smallest of perceptible sound intensities, the eardrum vibrates less distance than the diameter of a hydrogen atom! Well, it's so small an amount of power that you can hardly conceive of it, but if you have pretty good hearing, you could detect a sound wave with that small amount of power. That's not all. You could also detect a sound wave a thousand times more powerful, a million times more powerful, and even a billion times more powerful. And, that's before it even starts to get painful!

Figure 1.2: The ear is an astonishing receptor for sound waves. At the smallest of perceptible sound intensities, the eardrum vibrates less distance than the diameter of a hydrogen atom! If the energy in a single 1-watt night-light were converted to acoustical energy and divided up into equal portions for every person in the world, it would still be audible to the person with normal hearing.

I have a vivid fifth grade memory of my good friend, Norman. Norman was blind and the first and only blind person I ever knew well. I sat next to him in fifth grade and watched amazed as he banged away on his Braille typewriter. I would ask him questions about what he thought colors looked like and if he could explain the difference between light and dark. He would try to educate me about music beyond top-40 Pop, because he appreciated and knew a lot about jazz. But when it came to recess, we parted and went our separate ways – me to the playground and him to the wall outside the classroom. No one played with Norman. He couldn't see and so there was nothing for him. About once a day I would look over at Norman from high up on a jungle gym of bars and he would be smacking one of those rubbery creepy crawlers against the wall. He would do it all recess … every recess. I still marvel at how much Norman could get out of a simple sound. He didn't have sight so he had to compensate with his other senses. He got so much out of what I would have considered a very simplistic sound. For him the world of sound was rich and diverse and full. When I think of sound, I always think first of Norman. He's helped me to look more deeply and to understand how sophisticated the world of sound really is.

What about when more than one wave is present in the same place? For example, how is it that you can be at a symphony and make out the sounds of individual instruments while they all play together and also hear and understand a message being whispered to you at the same time you detect someone coughing five rows back? How do the sound waves combine to give you the totality as well as the individuality of each of the sounds in a room? These are some of the questions we will answer as we continue to pursue an understanding of music and musical instruments.

TWO TYPES OF WAVES

Waves come in two basic types, depending on their type of vibration. Imagine laying a long slinky on the ground and shaking it back and forth. You would make what is called a **transverse wave** (see Figure 1.3). A transverse wave is one in which the medium vibrates at right angles to the direction the energy moves. If instead, you pushed forward and pulled backward on the slinky you would make a **compressional wave** (see Figure 1.4). Compressional waves are also known as longitudinal waves. A compressional wave is one in which the medium vibrates in the same direction as the movement of energy in the wave

Certain terms and ideas related to waves are easier to visualize with transverse waves, so let's start by thinking about the transverse wave you could make with a slinky. Imagine taking a snapshot of the wave from the ceiling. It would look like Figure 1.5. Some wave vocabulary can be taken directly from the diagram. Other vocabulary must be taken from a mental image of the wave in motion:

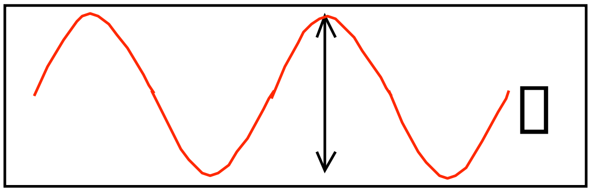


Figure 1.3: A transverse wave moves to the right while the medium vibrates at right angles, up and down.

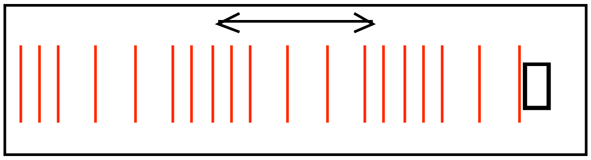


Figure 1.4: A compressional wave moves to the right while the medium vibrates in the same direction, right to left.

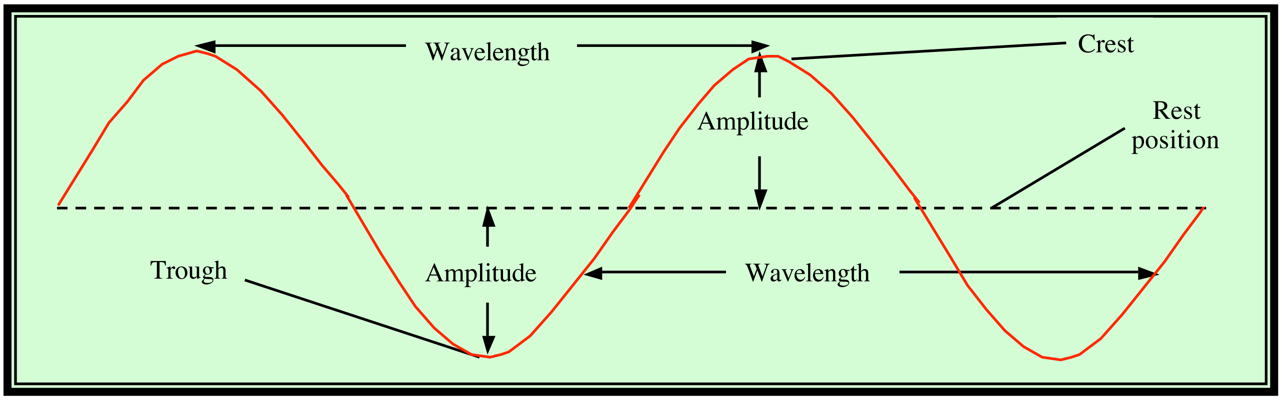


Figure 1.5: Wave vocabulary

CREST: The topmost point of the wave medium or greatest positive distance from the rest position.

TROUGH: The bottommost point of the wave medium or greatest negative distance from the rest position.

WAVELENGTH (□): The distance from crest to adjacent crest or from trough to adjacent trough or from any point on the wave medium to the adjacent corresponding point on the wave medium.

AMPLITUDE (A): The distance from the rest position to either the crest or the trough. The amplitude is related to the energy of the wave. As the energy grows, so does the amplitude. This makes sense if you think about making a more energetic slinky wave. You'd have to swing it with more intensity, generating larger amplitudes. The relationship is not linear though. The energy is actually proportional to the square of the amplitude. So a wave with amplitude twice as large actually has four times more energy and one with amplitude three times larger actually has nine times more energy.

The rest of the vocabulary requires getting a mental picture of the wave being generated. Imagine your foot about halfway down the distance of the slinky's stretch. Let's say that three wavelengths pass your foot each second.

FREQUENCY (f): The number of wavelengths to pass a point per second. In this case the frequency would be 3 per second. Wave frequency is often spoken of as "waves per second," "pulses per second," or "cycles per second." However, the SI unit for frequency is the Hertz (Hz). 1 Hz = 1 per second, so in the case of this illustration, $f = 3$ *Hz*.

PERIOD (T): The time it takes for one full wavelength to pass a certain point. If you see three wavelengths pass your foot every second, then the time for each wavelength to pass is $\frac{1}{3}$ of a second. Period and frequency are reciprocals of each other:

$$T = \frac{1}{f} \quad \text{and} \quad f = \frac{1}{T}$$

SPEED (v) Average speed is always a ratio of distance to time, $v = d/t$. In the case of wave speed, an appropriate distance would be wavelength, λ. The corresponding time would then have to be the period, T. So the wave speed becomes:

$$v = \frac{\lambda}{t} \quad \text{or} \quad v = \lambda f$$

SOUND WAVES

If a tree falls in the forest and there's no one there to hear it, does it make a sound? It's a common question that usually evokes a philosophical response. I could argue yes or no convincingly. You will too later on. Most people have a very strong opinion one way or the other. The problem is that their opinion is usually not based on a clear understanding of what sound is.

I think one of the easiest ways to understand sound is to look at something that has a simple mechanism for making sound. Think about how a tuning fork makes sound. Striking one of the forks causes you to immediately hear a tone. The tuning fork begins to act somewhat like a playground swing. The playground swing, the tuning fork, and most physical systems will act to restore themselves if they are stressed from their natural state. The "natural state" for the swing, is to hang straight down. If you push it or pull it and then let go, it moves back towards the position of hanging straight down. However, since it's moving when it reaches that point, it actually overshoots and, in effect, stresses itself. This causes another attempt to restore itself and the movement continues back and forth until friction and air resistance have removed all the original energy of the push or pull. The same is true for the tuning fork. It's just that the movement (amplitude) is so much smaller that you can't visibly see it. But if you touched the fork you could feel it. Indeed, every time the fork moves back and forth it smacks the air in its way. That smack creates a small compression of air molecules that travels from that point as a compressional wave. When it reaches your ear, it vibrates your eardrum with the same frequency as the frequency of the motion of the tuning fork. You mentally process this vibration as a specific tone.

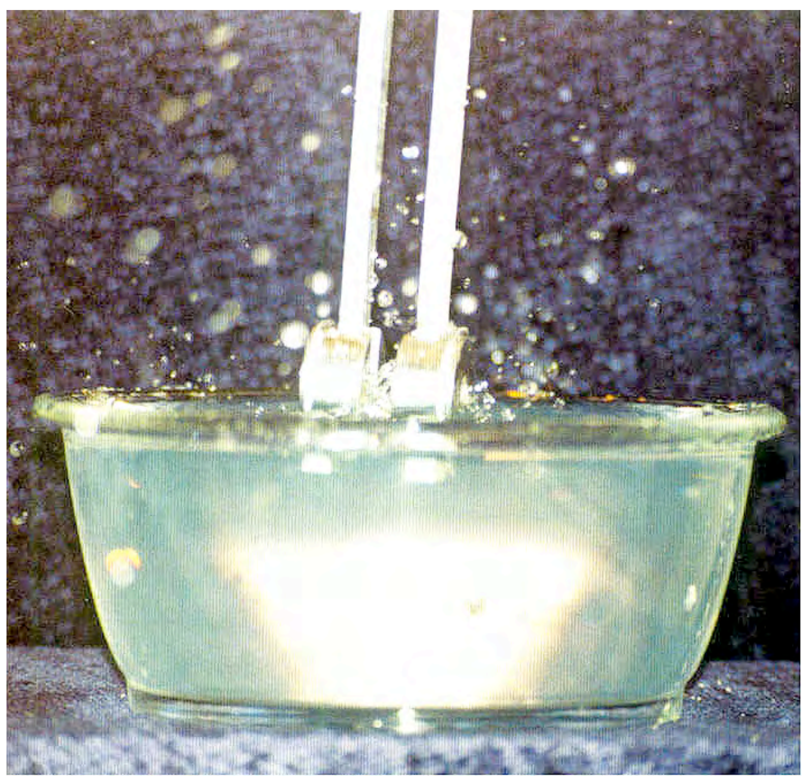


Figure 1.6: The energy of the vibrating tuning fork violently splashes water from the bowl. In the air, the energy of the tuning fork is transmitted through the air and to the ears.

Figure 1.7: The front of a speaker cone faces upward with several pieces of orange paper lying on top of it. Sound is generated when an electric signal causes the speaker cone to move in and out, pushing on the air and creating a compressional wave. The ear can detect these waves. Here these vibrations can be seen as they cause the little bits of paper to dance on the surface of the speaker cone.

A sound wave is nothing more than a compressional wave caused by vibrations. Next time you have a chance, gently feel the surface of a speaker cone (see Figure 1.7). The vibrations you feel with your fingers are the same vibrations disturbing the air. These vibrations eventually relay to your ears the message that is being broadcast. So, if a tree falls in the forest and there's no one there to hear it, does it make a sound? Well … yes, it will certainly cause vibrations in the air and ground when it strikes the ground. And … no, if there's no one there to mentally translate the vibrations into tones, then there can be no true sound. You decide. Maybe it is a philosophical question after all.

CHARACTERIZING SOUND

All sound waves are compressional waves caused by vibrations, but the music from a symphony varies considerably from both a baby's cry and the whisper of a confidant. All sound waves can be characterized by their **speed**, by their **pitch**, by their **loudness**, and by their **quality or timbre**.

The **speed of sound** is fastest in solids (almost 6000 m/s in steel), slower in liquids (almost 1500 m/s in water), and slowest in gases. We normally listen to sounds in air, so we'll look at the speed of sound in air most carefully. In air, sound travels at:

$$v = 331\tfrac{m}{s} + \left(0.6\frac{m/s}{°C}\right)Temperature$$

The part to the right of the "+" sign is the temperature factor. It shows that the speed of sound increases by 0.6 m/s for every temperature increase of 1°C. So, at 0° C, sound travels at 331 m/s (about 740 mph). But at room temperature (about 20°C) sound travels at:

$$\begin{aligned} v &= 331\frac{m}{s} + \left(0.6\frac{m/s}{°C}\right)(20°C) \\ &= 343\frac{m}{s} \end{aligned}$$

(This is the speed you should assume if no temperature is given).

It was one of aviation's greatest accomplishments when Chuck Yeager, on Oct. 14, 1947, flew his X-1 jet at Mach 1.06, exceeding the speed of sound by 6%. Regardless, this is a snail's pace compared to the speed of light. Sound travels through air at about a million times slower than light, which is the reason why we hear sound echoes but don't see light echoes. It's also the reason we see the lightning before we hear the thunder. The lightning-thunder effect is often noticed in big stadiums. If you're far away from a baseball player who's up to bat, you can clearly see the ball hit before you hear the crack of the bat. You can consider that the light recording the event reaches your eyes virtually instantly. So if the sound takes half a second more time than the light, you're half the distance sound travels in one second (165 meters) from the batter. Next time you're in a thunderstorm use this method to estimate how far away lightning is striking. Click here for a demonstration of the effect of echoes.

Do you get it? (1.1)

Explain why some people put their ears on railroad tracks in order to hear oncoming trains

> ### *Do you get it? (1.2)*
> The echo of a ship's foghorn, reflected from the cliff on a nearby island, is heard 5.0 s after the horn is sounded. How far away is the cliff?

The **pitch of sound** is the same as the frequency of a sound wave. With no hearing losses or defects, the ear can detect wave frequencies between 20 Hz and 20,000 Hz. (Sounds below 20 Hz are classified as subsonic; those over 20,000 Hz are ultrasonic). However, many older people, loud concert attendees, and soldiers with live combat experience lose their ability to hear higher frequencies. The good news is that the bulk of most conversation takes place well below 2,000 Hz. The average frequency range of the human voice is 120 Hz to approximately 1,100 Hz (although a baby's shrill cry is generally 2,000 - 3,000 Hz – which is close to the frequency range of greatest sensitivity … hmm, interesting). Even telephone frequencies are limited to below 3,400 Hz. But the bad news is that the formation of many consonants is a complex combination of very high frequency pitches. So to the person with high frequency hearing loss, the words key, pee, and tea sound the same. You find people with these hearing losses either lip reading or understanding a conversation by the context as well as the actual recognition of words. Neil Bauman, a hearing expert at www.hearinglosshelp.com, offers the following information:

> *"Vowels are clustered around the frequencies between about 300 and 750 Hz. Some consonants are very low frequency, such as j, z, and v – at about 250 Hz. Others such as k, t, f, s, and th are high frequency sounds and occur between 3,000 and 8,000 Hz. All of these consonants are voiceless ones and are produced by air hissing from around the teeth. Most people, as they age, begin losing their hearing at the highest frequencies first and progress downwards. Thus, the above consonant sounds are the first to be lost. As a result, it is most difficult to distinguish between similar sounding words that use these letters. Furthermore, the vowels are generally loud (they use about 95% of the voice energy to produce). The consonants are left with only 5% to go around for all of them. But it is mostly the consonants that give speech its intelligibility. That is why many older people will say, 'I can hear people talking. I just can't understand what they are saying.'"*

One important concept in music is the *octave* – a doubling in frequency. For example, 40 Hz is one octave higher than 20 Hz. The ear is sensitive over a frequency range of about 10 octaves: 20 Hz → 40 Hz → 80 Hz → 160 Hz → 320 Hz → 640 Hz → 1,280 Hz → 2,560 Hz → 5,120 Hz → 10,240 Hz → 20,480 Hz. And within that range it can discriminate between thousands of differences in sound frequency. Below about 1,000 Hz the **Just Noticeable Difference** (JND) in frequency is about 1 Hz (at the loudness at which most music is played), but this rises sharply beyond 1,000 Hz. At 2,000 the JND is about 2 Hz and at 4,000 Hz the JND is about 10 Hz. (A JND of 1 Hz at 500 Hz means that if you were asked to listen alternately to tones of 500 Hz and 501 Hz, the two could be distinguished as two different frequencies, rather than the same). It is interesting to compare the ear's frequency perception to that of the eye. From red to violet, the frequency of light less than doubles, meaning that the eye is only sensitive over about one octave, and its ability to discriminate between different colors is only about 125. The ear is truly an amazing receptor, not only its frequency range, but also in its ability to accommodate sounds with vastly different loudness.

The **loudness of sound** is related to the amplitude of the sound wave. Most people have some recognition of the decibel (dB) scale. They might be able to tell you that 0 dB is the threshold of hearing and that the sound on the runway next to an accelerating jet is about 140 dB. However, most people don't realize that the decibel scale is a logarithmic scale. This means that for every increase of 10 dB the sound intensity increases by a factor of ten. So going from 60 dB to 70 dB is a ten-fold increase, and 60 dB to 80 dB is a hundred-fold increase. This is amazing to me. It means that we can hear sound intensities over 14 orders of magnitude. This means that the 140 dB jet on the runway has a loudness of 10^{14} times greater than threshold. 10^{14} is 100,000,000,000,000 – that's 100 trillion! It means our ears can measure loudness over a phenomenally large range. Imagine having a measuring cup that could accurately measure both a teaspoon and 100 trillion teaspoons (about 10 billion gallons). The ear is an amazing receptor! However, our perception is skewed a bit. A ten-fold increase in loudness doesn't sound ten times louder. It may only sound twice as loud. That's why when cheering competitions are done at school rallies, students are not very excited by the measure of difference in loudness between a freshmen class (95 dB) and a senior class (105 dB). The difference is only 10 dB. It sounds perhaps twice as loud, but it's really 10 times louder. (Those lungs *and confidence* grow a lot in three years!)

Do you get it? (1.3)

When a passenger at the airport moves from inside a waiting area to outside near an airplane the decibel level goes from 85 dB to 115 dB. By what factor has the sound intensity actually gone up?

The **quality of sound** or **timbre** is the subtlest of all its descriptors. A trumpet and a violin could play exactly the same note, with the same pitch and loudness, and if your eyes were closed you could easily distinguish between the two. The difference in the sounds has to do with their *quality* or *timbre*. The existence of supplementary tones, combined with the basic tones, doesn't change the basic pitch, but gives a special "flavor" to the sound being produced. Sound quality is the characteristic that gives the identity to the sound being produced.

DETAILS ABOUT DECIBELS

It was mentioned earlier that the sensitivity of the human ear is phenomenally keen. The threshold of hearing (what a young perfect ear could hear) is $1\times10^{-12}\,Watts/meter^2$. This way of expressing sound wave amplitude is referred to as *Sound Intensity* (*I*). It is not to be confused with *Sound Intensity Level* (*L*), measured in decibels (dB). The reason why loudness is routinely represented in decibels rather than $Watts/meter^2$ is primarily because the ears don't hear linearly. That is, if the sound intensity doubles, it doesn't sound twice as loud. It doesn't really sound twice as loud until the Sound Intensity is about ten times greater. (This is a very rough approximation that depends on the frequency of the sound as well as the reference intensity.) If the sound intensity were used to measure loudness, the scale would have to span 14 orders of magnitude. That means that if we defined the faintest sound as "1", we would have to use a scale that went up to 100,000,000,000,000 (about the loudest sounds you ever hear. The decibel scale is much more compact (0 dB – 140 dB for the same range) and it is more closely linked to our ears' perception of loudness. You can think of the *sound intensity* as a physical measure of sound wave amplitude and *sound intensity level* as its psychological measure.

The equation that relates sound intensity to sound intensity level is:

$$L = 10\log\frac{I_2}{I_1}$$

$L \equiv$ The number of decibels I_2 is greater than I_1
$I_2 \equiv$ The higher sound intensity being compared
$I_1 \equiv$ The lower sound intensity being compared

Remember, I is measured in $Watts/meter^2$. It is like the raw power of the sound. The L in this equation is what the decibel difference is between these two. In normal use, I_1 is the threshold of hearing, $1\times10^{-12}\,Watts/meter^2$. This means that the decibel difference is with respect to the faintest sound that can be heard. So when you hear that a busy intersection is 80 dB, or a whisper is 20 dB, or a class cheer is 105 dB, it always assumes that the comparison is to the threshold of hearing, 0 dB. ("80 dB" means 80 dB greater than threshold). Don't assume that 0 dB is no sound or total silence. This is simply the faintest possible sound a human with perfect hearing can hear. Table 1.1 provides decibel levels for common sounds.

If you make I_2 twice as large as I_1, then $\Delta L \approx 3dB$. If you make I_2 ten times as large as I_1, then $\Delta L = 10dB$. These are good reference numbers to tuck away:

$$Double\ \ Sound\ \ Intensity \Rightarrow +3dB$$

$$10\times\ \ Sound\ \ Intensity = +10dB$$

Click here to listen to a sound intensity level reduced by 6 dB per step over ten steps. Click here to listen to a sound intensity level reduced by 3 dB per step over ten steps.

Source of sound	Sound Intensity Level (dB)	Sound Intensity $\left(\frac{W}{m^2}\right)$
Threshold of hearing	0	1×10^{-12}
Breathing	20	1×10^{-10}
Whispering	40	1×10^{-8}
Talking softly	60	1×10^{-6}
Loud conversation	80	1×10^{-4}
Yelling	100	1×10^{-2}
Loud Concert	120	1
Jet takeoff	140	100

Table 1.1 Decibel levels for typical sounds

FREQUENCY RESPONSE OVER THE AUDIBLE RANGE

We hear lower frequencies as low pitches and higher frequencies as high pitches. However, our sensitivity varies tremendously over the audible range. For example, a 50 Hz sound must be 43 dB before it is perceived to be as loud as a 4,000 Hz sound at 2 dB. (4,000 Hz is the approximate frequency of greatest sensitivity for humans with no hearing loss.) In this case, we require the 50 Hz sound to have 13,000 times the actual intensity of the 4,000 Hz sound in order to have the same *perceived* intensity! Table 1.2 illustrates this phenomenon of sound intensity level versus frequency. The last column puts the relative intensity of 4,000 Hz arbitrarily at 1 for easy comparison with sensitivity at other frequencies.

If you are using the CD version of this curriculum you can try the following demonstration, which illustrates the response of the human ear to frequencies within the audible range Click here to calibrate the sound on your computer and then click here for the demonstration.

Frequency (Hz)	Sound Intensity Level (dB)	Sound Intensity $\left(\frac{W}{m^2}\right)$	Relative Sound Intensity
50	43	2.0×10^{-8}	13,000
100	30	1.0×10^{-9}	625
200	19	7.9×10^{-11}	49
500	11	1.3×10^{-11}	8.1
1,000	10	1.0×10^{-11}	6.3
2,000	8	6.3×10^{-12}	3.9
3,000	3	2.0×10^{-12}	1.3
4,000	2	1.6×10^{-12}	1
5,000	7	5.0×10^{-12}	3.1
6,000	8	6.3×10^{-12}	3.9
7,000	11	1.3×10^{-11}	8.1
8,000	20	1.0×10^{-10}	62.5
9,000	22	1.6×10^{-10}	100
14,000	31	1.3×10^{-9}	810

Table 1.2: Sound intensity and sound intensity level required to perceive sounds at different frequencies to be equally loud. A comparison of relative sound intensities arbitrarily assigns 4,000 Hz the value of 1.

Example

The muffler on a car rusts out and the decibel level increases from 91 dB to 113 dB. How many times louder is the leaky muffler?

The "brute force" way to do this problem would be to start by using the decibel equation to calculate the sound intensity both before and after the muffler rusts out. Then you could calculate the ratio of the two. It's easier though to recognize that the decibel difference is 22 dB and use that number in the decibel equation to find the ratio of the sound intensities directly:

$$L = 10\log\frac{I_2}{I_1}$$

$$22dB = 10\log\frac{I_2}{I_1} \Rightarrow 2.2 = \log\frac{I_2}{I_1}$$

Notice I just dropped the dB unit. It's not a real unit, just kind of a placeholder unit so that we don't have to say, "The one sound is 22 more than the other sound." and have a strange feeling of "22 … *what*?"

$$10^{2.2} = \frac{I_2}{I_1} \Rightarrow I_2 = 10^{2.2} I_1 \Rightarrow I_2 = 158 I_1$$

So the muffler is actually 158 times louder than before it rusted out.

Do you get it? (1.4)

A solo trumpet in an orchestra produces a sound intensity level of 84 dB. Fifteen more trumpets join the first. How many decibels are produced?

Do you get it? (1.5)

What would it mean for a sound to have a sound intensity level of -10 dB

Another factor that affects the intensity of the sound you hear is how close you are to the sound. Obviously a whisper, barely detected at one meter could not be heard across a football field. Here's the way to think about it. The power of a particular sound goes out in all directions. At a meter away from the source of sound, that power has to cover an area equal to the area of a sphere ($4\pi r^2$) with a radius of one meter. That area is 4π m^2. At two meters away the same power now covers an area of $4\pi(2\ m)^2 = 16\pi\ m^2$, or four times as much area. At three meters away the same power now covers an area of $4\pi(3\ m)^2 = 36\pi\ m^2$, or nine times as much area. So compared to the intensity at one meter, the intensity at two meters will be only one-quarter as much and the intensity at three meters only one-ninth as much. The sound intensity follows an **inverse square** law, meaning that by whatever factor the distance from the source of sound changes, the intensity will change by the square of the reciprocal of that factor.

Example

If the sound intensity of a screaming baby were $1\times10^{-2}\ \frac{W}{m^2}$ at 2.5 m away, what would it be at 6.0 m away?

The distance from the source of sound is greater by a factor of $\frac{6.0}{2.5} = 2.4$. So the sound intensity is decreased by $\frac{1}{(2.4^2)} = 0.174$. The new sound intensity is:

$$\left(1\times10^{-2}\ \frac{W}{m^2}\right)(0.174) = \boxed{1.74\times10^{-3}\ \frac{W}{m^2}}$$

Another way to look at this is to first consider that the total power output of a source of sound is its sound intensity in $Watts/meter^2$ multiplied by the area of the sphere that the sound has reached. So, for example, the baby in the problem above creates a sound intensity of $1\times10^{-2} W/m^2$ at 2.5 m away. This means that the total power put out by the baby is:

$$Power = Intensity \times sphere\ \ area$$

$$\Rightarrow\ P = \left[1\times10^{-2}\ \frac{W}{m^2}\right]\left[4\pi(2.5m)^2\right] = 0.785\ \ W$$

Now let's calculate the power output from the information at 6.0 m away:

$$P = \left[1.74\times10^{-3}\ \frac{W}{m^2}\right]\left[4\pi(6.0m)^2\right] = .785\ \ W$$

It's the same of course, because the power output depends on the baby, not the position of the observer. This means we can always equate the power outputs that are measured at different locations:

$$P_1 = P_2\ \Rightarrow\ (I_1)(4\pi r_1^2) = (I_2)(4\pi r_2^2)$$

$$\Rightarrow\ I_1 r_1^2 = I_2 r_2^2$$

Do you get it? (1.6)

It's a good idea to make sure that you keep the chronic sound you're exposed to down under 80 dB. If you were working 1.0 m from a machine that created a sound intensity level of 92 dB, how far would you need move away to hear only 80 dB? (Hint: remember to compare sound intensities and not sound intensity levels.)

ACTIVITY

ORCHESTRAL SOUND

You can hear plenty of sound in a concert hall where an orchestra is playing. Each instrument vibrates in its own particular way, producing the unique sound associated with it. The acoustical power coming from these instruments originates with the musician. It is the energy of a finger thumping on a piano key and the energy of the puff of air across the reed of the clarinet and the energy of the slam of cymbals against each other that causes the instrument's vibration. Most people are surprised to learn that only about 1% of the power put into the instrument by the musician actually leads to the sound wave coming from the instrument. But, as you know, the ear is a phenomenally sensitive receptor of acoustical power and needs very little power to be stimulated to perception. Indeed, the entire orchestra playing at once would be loud to the ear, but actually generate less power than a 75-watt light bulb! An orchestra with 75 performers has an acoustic power of about 67 watts. To determine the sound intensity level at 10 m, we could start by finding the sound intensity at 10 m:

$$I = \frac{67W}{4\pi(10m)^2} = 0.056\frac{W}{m^2}.$$

The sound intensity level would then be:

$$L = 10\log\left(\frac{0.056\frac{W}{m^2}}{1\times10^{-12}\frac{W}{m^2}}\right) = 107.5dB.$$

Table 1.3 lists the sound intensity level of various instruments in an orchestra as heard from 10 m away. You can use these decibel levels to answer the questions throughout this activity.

Orchestral Instrument	**Sound Intensity Level (*dB*)**
Violin (at its quietest)	34.8
Clarinet	76.0
Trumpet	83.9
Cymbals	98.8
Bass drum (at its loudest)	103

Table 1.3: Sound intensity levels (measured at 10 m away) for various musical instruments

1. How many clarinets would it take to equal the acoustic power of a pair of cymbals?

2. If you had to replace the total acoustic power of a bass drum with a single light bulb, what wattage would you choose?

3. How far would you have to be from the violin (when it's at its quietest) in order to barely detect its sound?

4. If the sound emerges from the trumpet at 0.5 m from the trumpet player's ear, how many decibels does he experience during his trumpet solo?

5. If the orchestra conductor wanted to produce the sound intensity of the entire orchestra, but use only violins (at their quietest) to produce the sound, how many would need to be used? How about if it were to be done with bass drums (at their loudest?

6. The conductor has become concerned about the high decibel level and wants to make sure he does not experience more than 100 *dB*. How far away from the orchestra must he stand?

7. If he doesn't use the one bass drum in the orchestra, how far away does he need to stand?

WAVE INTERFERENCE

IT'S INTRIGUING THAT at a lively party with everyone talking at once, you can hear the totality of the "noise" in the room and then alternately distinguish and concentrate on the conversation of one person. That is because of an interesting phenomenon of waves called **superposition**. Wave superposition occurs when two or more waves move through the same space in a wave medium together. There are two important aspects of this wave superposition. One is that each wave maintains its own identity and is unaffected by any other waves in the same space. This is why you can pick out an individual conversation among all the voices in the region of your ear. The second aspect is that when two or more waves are in the same medium, the overall amplitude at any point on the medium is simply the sum of the individual wave amplitudes at that point. Figure 1.8 illustrates both of these aspects. In the top scene, two wave pulses move toward each other. In the second scene the two pulses have reached the same spot in the medium and the combined amplitude is just the sum of the two. In the last scene, the two wave pulses move away from each other, clearly unchanged by their meeting in the second scene.

When it comes to music, the idea of interference is exceptionally important. Musical sounds are often constant frequencies held for a sustained period. Sound waves interfere in the same way other waves, but when the sound waves are musical sounds (sustained constant pitches), the resulting superposition can sound either pleasant (consonant) or unpleasant (dissonant). Musical scales consist of notes (pitches), which when played together, sound consonant. We'll use the idea of sound wave interference when we begin to look for ways to avoid dissonance in the building of musical scales.

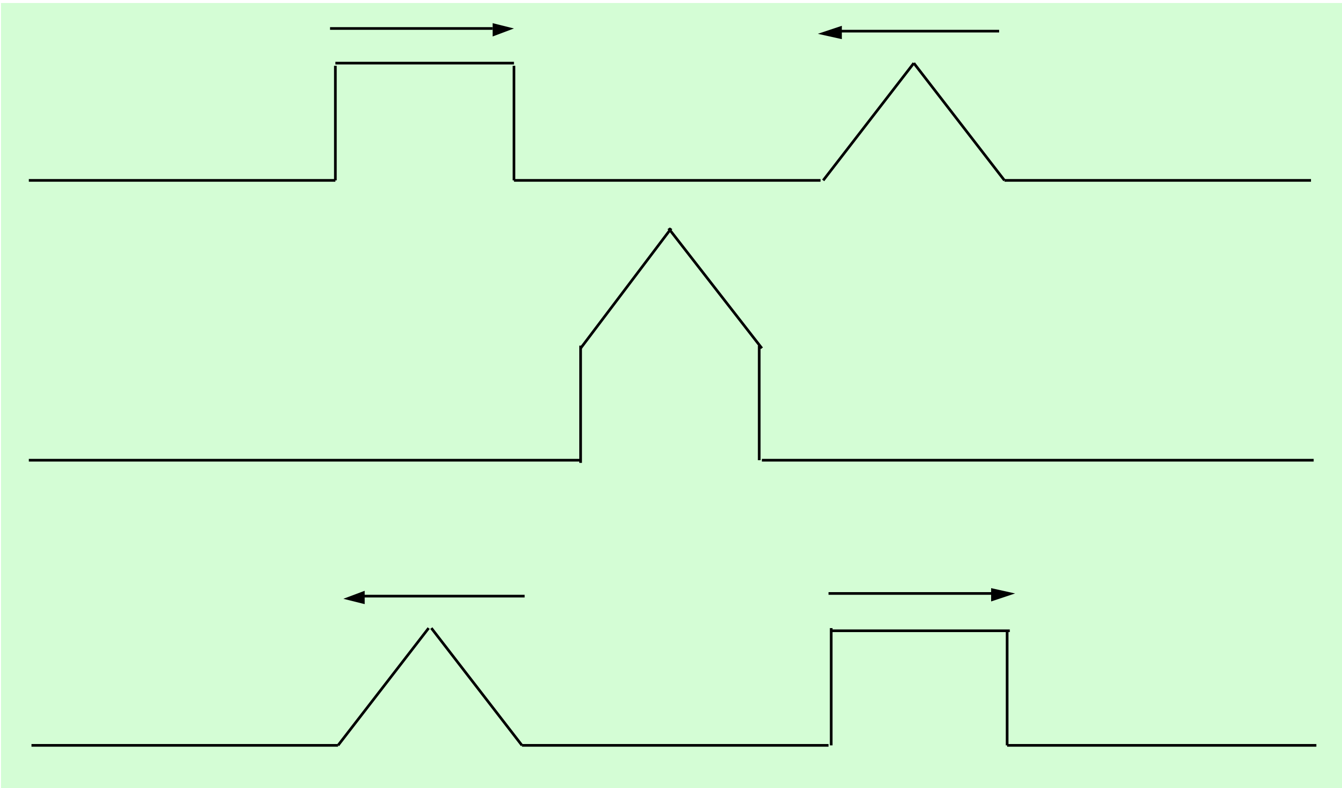
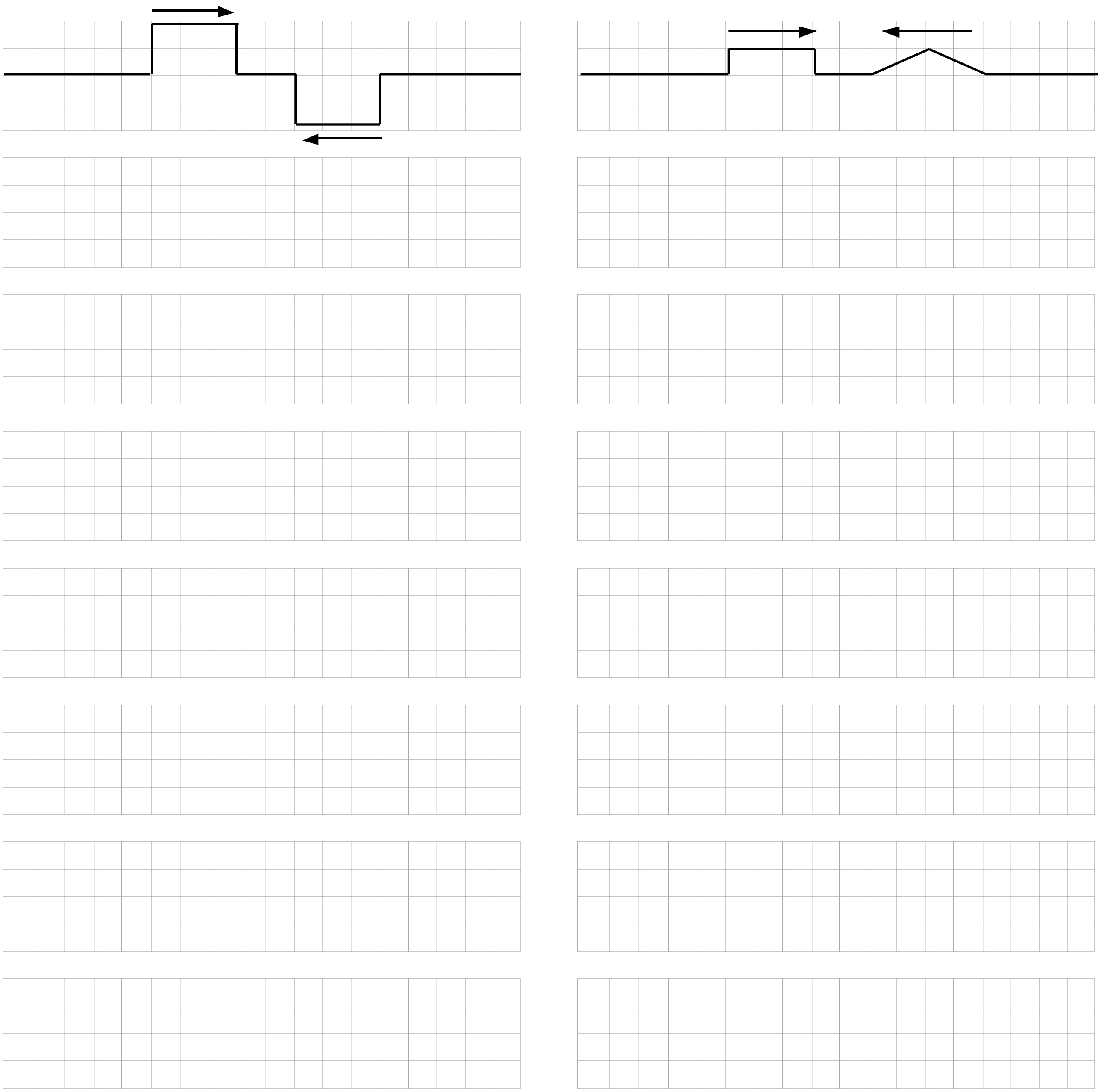


Figure 1.8: Wave superposition. Note in the middle drawing that the wave shape is simply the arithmetic sum of the amplitudes of each wave. Note also in the bottom drawing that the two waves have the same shape and amplitude as they had before encountering each other.

ACTIVITY

WAVE INTERFERENCE

In each of the following two cases, the wave pulses are moving toward each other. Assume that each wave pulse moves one graph grid for each new graph. Draw the shape the medium would have in each of the blank graphs below.



"Music should strike fire from the heart of man, and bring tears from the eyes of woman."
– Ludwig Van Beethoven

CHAPTER 2
RESONANCE, STANDING WAVES, AND MUSICAL INSTRUMENTS

THE NEXT STEP in pursuing the physics of music and musical instruments is to understand how physical systems can be made to vibrate in frequencies that correspond to the notes of the musical scales. But before the vibrations of physical systems can be understood, a diversion to the behavior of waves must be made once again. The phenomena of **resonance** and **standing waves** occur in the structures of all musical instruments. It is the means by which all musical instruments … make their music.

Why is it that eight-year-old boys have such an aversion to taking baths? I used to hate getting in the tub! It was perhaps my least favorite eight-year-old activity. The one part of the bathing ritual that made the whole process at least tolerable was … making waves! If I sloshed back and forth in the water with my body at just the right frequency, I could create waves that reached near tidal wave amplitudes. Too low or too high a frequency and the waves would die out. But it was actually easy to find the right frequency. It just *felt* right. I thought I had discovered something that no one else knew. Then, 20 years later, in the pool at a Holiday Inn in New Jersey with a group of other physics teachers, I knew that my discovery was not unique. Lined up on one side of the pool and with one group movement, we heaved our bodies forward. A water wave pulse moved forward and struck the other side of the pool. Then it reflected and we waited as it traveled back toward us, continued past us, and reflected again on the wall of the pool right behind us. Without a word, as the crest of the doubly reflected wave reached us, we heaved our bodies again. Now the wave pulse had greater amplitude. We had added to it and when it struck the other side, it splashed up on the concrete, drawing the amused and, in some cases, irritated attention of the other guests sunning themselves poolside. As a child pushes her friend on a playground swing at just right time in order to grow the amplitude of the swing's motion, we continued to drive the water wave amplitude increasingly larger with our rhythmic motion. Those who were irritated before were now curious, and those who were amused before were now cheering. The crowd was pleased and intrigued by something they knew in their gut. Most had probably rocked back and forth on the seat of a car at a stoplight with just the right frequency to get the entire car visibly rocking. Many had probably had the experience of grabbing a sturdy street sign and pushing and pulling on it at just the right times to get the sign to shake wildly. They had all experienced this phenomenon of **resonance**.

To understand resonance, think back to the discussion of the playground swing and tuning fork restoring themselves to their natural states after being stressed out of those states. Recall that as the swing moves through the bottom of its motion, it overshoots this natural state. The tuning fork does too, and both of the two will oscillate back and forth past this point (at a natural frequency particular to the system) until the original energy of whatever stressed them is dissipated. You can keep the swing moving and even increase its amplitude by pushing on it at this same natural frequency. The same could be done to the tuning fork if it were driven with an audio speaker producing the fork's natural frequency. Resonance occurs whenever a physical system is driven at its natural frequency. Most physical systems have many natural frequencies. The street sign, for example can be made to shake wildly back and forth with a low **fundamental frequency** (**first mode**) or with a higher frequency of vibration, in the **second mode**. It's easier to understand how musical instruments can be set into resonance by thinking about **standing waves**.

Standing waves occur whenever two waves with equal frequency and wavelength move through a medium so that the two perfectly reinforce each other.

In order for this to occur the length of the medium must be equal to some integer multiple of half the wavelength of the waves. Usually, one of the two waves is the reflection of the other. When they reinforce each other, it looks like the energy is *standing* in specific locations on the wave medium, hence the name *standing waves* (see figure 2.1). There are parts of the wave medium that remain motionless (the **nodes**) and parts where the wave medium moves back and forth between maximum positive and maximum negative amplitude (the **antinodes**).

Standing waves can occur in all wave mediums and can be said to be present during any resonance. Perhaps you've heard someone in a restaurant rubbing a finger around the rim of a wineglass, causing it to sing. The "singing" is caused by a standing wave in the glass that grows in amplitude until the pulses against the air become audible. Soldiers are told to "break step march" when moving across small bridges because the frequency of their march may be the natural frequency of the bridge, creating and then reinforcing a standing wave in the bridge and causing the same kind of resonance as in the singing wine glass. A standing wave in a flat metal plate can be created by driving it at one of its natural frequencies at the center of the plate. Sand poured onto the plate will be unaffected by and collect at the nodes of the standing wave, whereas sand at the antinodes will be bounced off, revealing an image of the standing wave (see figure 2.2).

Resonance caused the destruction of the Tacoma Narrows Bridge on November 7, 1940 (see Figure 2.3). Vortices created around its deck by 35 - 40 mph winds resulted in a standing wave in the bridge deck. When its amplitude reached five feet, at about 10 am, the bridge was forced closed. The amplitude of motion eventually reached 28 feet. Most of the remains of this bridge lie at the bottom of the Narrows, making it the largest man-made structure ever lost to sea and the largest man-made reef.

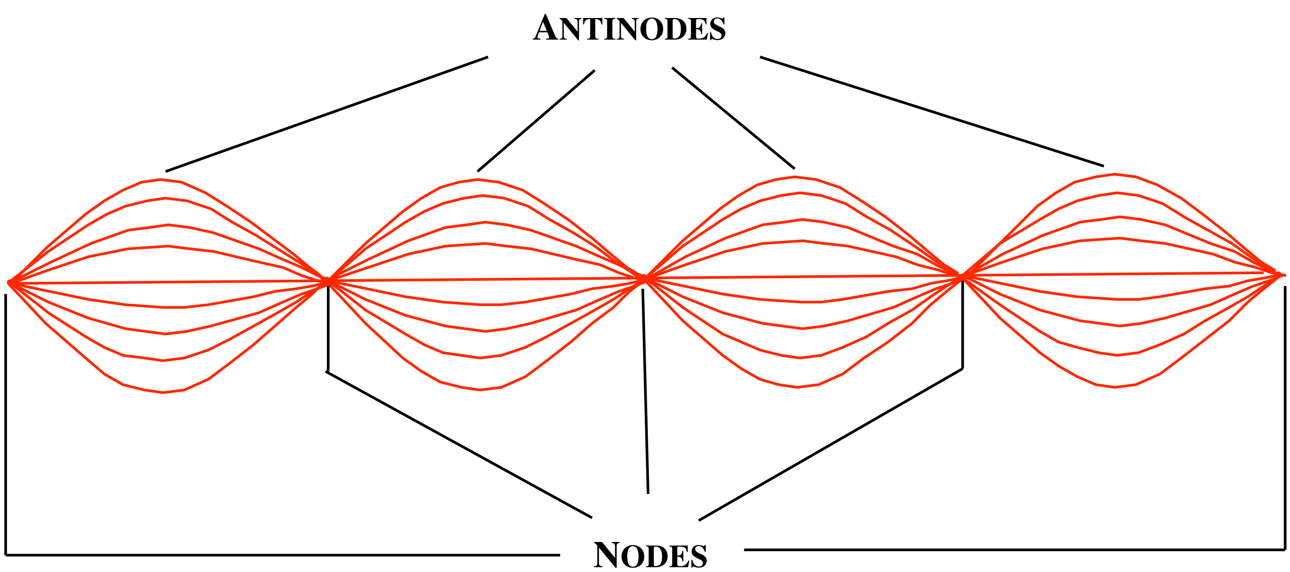


Figure 2.1: A time-lapse view of a standing wave showing the nodes and antinodes present. It is so named because it appears that the energy in the wave stands in certain places (the antinodes). Standing waves are formed when two waves of equal frequency and wavelength move through a medium and perfectly reinforce each other. In order for this to occur the length of the medium must be equal to some integer multiple of half the wavelength of the waves. In the case of this standing wave, the medium is two wavelengths long (4 half wavelengths).

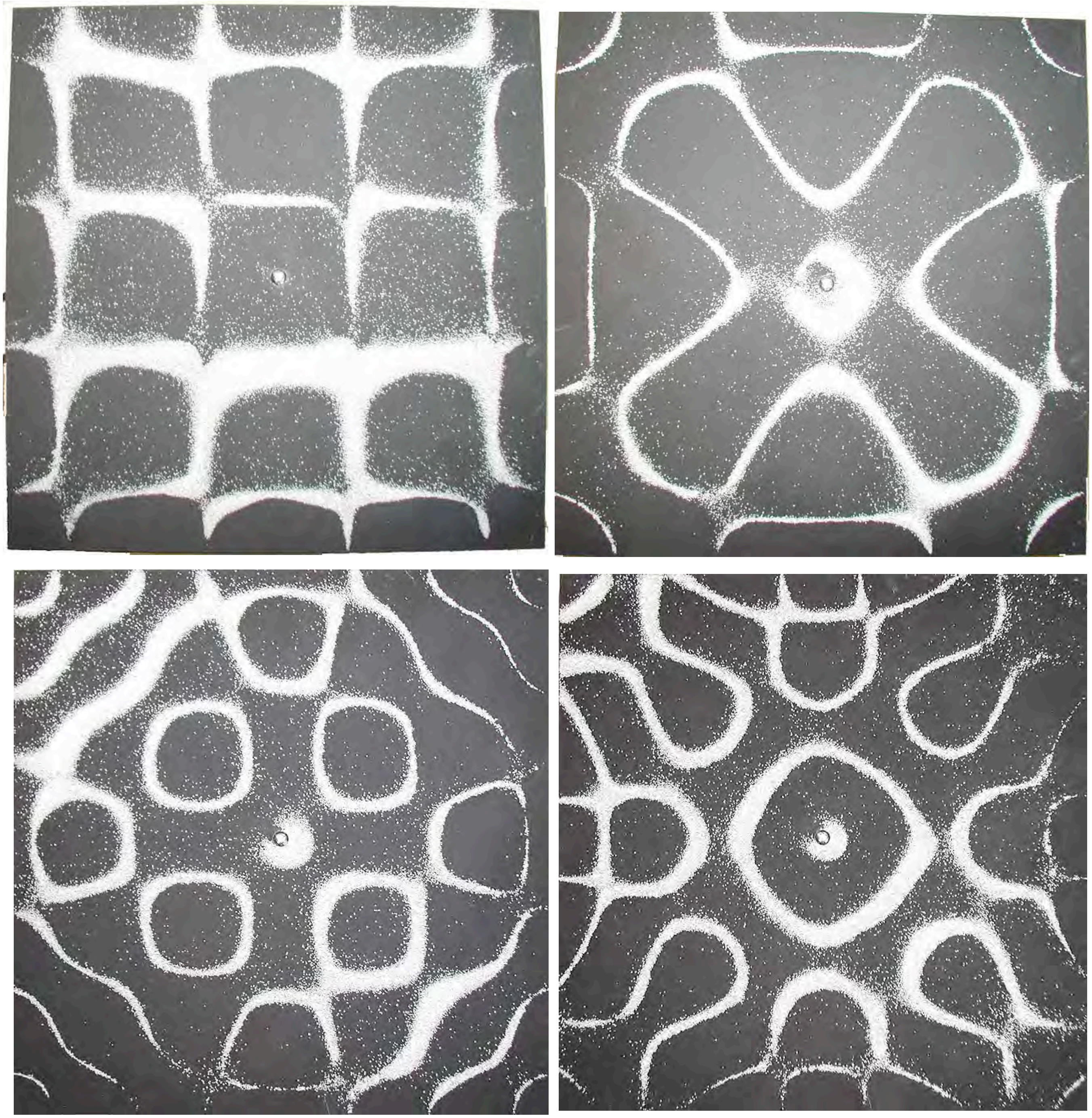
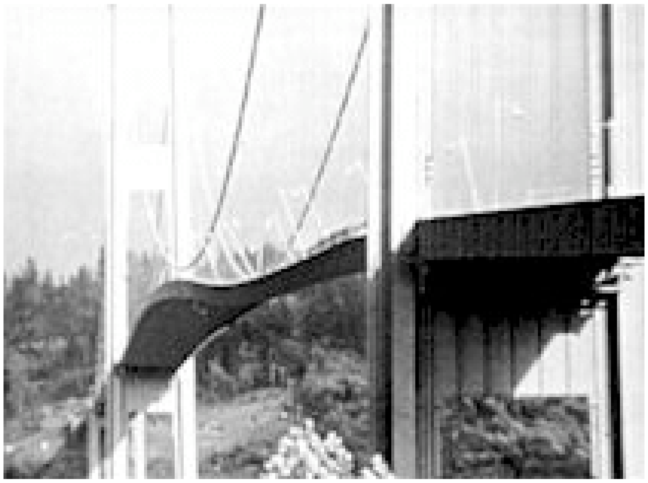


Figure 2.2: A flat square metal plate is driven in four of its resonance modes. These photographs show each of the resulting complicated two-dimensional standing wave patterns. Sand poured onto the top of the plate bounces off the antinodes, but settles into the nodes, allowing the standing wave to be viewed. The head of a drum, when beaten, and the body of a guitar, when played, exhibit similar behaviors.



The Tacoma Narrows Bridge on November 7, 1940. Workers on the construction of the bridge had referred to it as "Galloping Gertie" because of the unusual oscillations that were often present. Note nodes at the towers and in the center of the deck. This is the second mode.

Vortices around its deck caused by winds of 35 – 40 mph caused the bridge to begin rising and falling up to 5 feet, forcing the bridge to be closed at about 10 am. The amplitude of motion eventually reached 28 feet.

The bridge had a secondary standing wave in the first mode with its one node on the centerline of the bridge. The man in the photo wisely walked along the node. Although the bridge was heaving wildly, the amplitude at the node was zero, making it easy to navigate.

At 10:30 am the oscillations had finally caused the center span floor panel to fall from the bridge. The rest of the breakup occurred over the next 40 minutes.

Figure 2.3: The Tacoma Narrows Bridge collapse (used with permission from University of Washington Special Collection Manuscripts).

Since the collapse of the Tacoma Narrows Bridge, engineers have been especially conscious of the dangers of resonance caused by wind or earthquakes. Paul Doherty, a physicist at the Exploratorium in San Francisco, had this to say about the engineering considerations now made when building or retrofitting large structures subject to destruction by resonance:

"The real world natural frequency of large objects such as skyscrapers and bridges is determined via remote sensor accelerometers. Buildings like the Trans-America Pyramid and the Golden Gate Bridge have remote accelerometers attached to various parts of the structure. When wind or an earthquake 'excites' the building the accelerometers can detect the 'ringing' (resonant oscillation) of the structure. The Golden Gate Bridge was 'detuned' by having mass added at various points so that a standing wave of a particular frequency would affect only a small portion of the bridge. The Tacoma Narrows Bridge had the resonance extend the entire length of the span and only had a single traffic deck which could flex. The Golden Gate Bridge was modified after the Narrows Bridge collapse. The underside of the deck had stiffeners added to dampen torsion of the roadbed and energy-absorbing struts were incorporated. Then mass additions broke up the ability of the standing wave to travel across the main cables because various sections were tuned to different oscillation frequencies. This is why sitting in your car waiting to pay the toll you can feel your car move up a down when a large truck goes by but the next large truck may not give you the same movement. That first truck traveling at just the right speed may excite the section you are on while a truck of different mass or one not traveling the same speed may not affect it much. As for the horizontal motion caused by the wind, the same differentiation of mass elements under the roadbed keeps the whole bridge from going resonant with Æolian oscillations.

Just envision the classic Physics experiment where different length pendulums are hung from a common horizontal support. Measured periodic moving of the support will make only one pendulum swing depending on the period of the applied motion. If all the pendulums had the same oscillation you could get quite a motion going with a small correctly timed force application. Bridges and buildings now rely on irregular distributions of mass to help keep the whole structure from moving as a unit that would result in destructive failure. Note also on the Golden Gate the secondary suspension cable 'keepers' (spacers) are located at slightly irregular intervals to detune them. As current structural engineering progresses more modifications of the bridge will be done. The new super bridge in Japan has hydraulically movable weights that can act as active dampeners. What is earthquake (or wind) safe today will be substandard in the future."

The collapse of the Tacoma Narrows Bridge is perhaps the most spectacular example of the destruction caused by resonance, but everyday there are boys and girls who grab hold of street sign poles and shake them gently – intuitively – at just the right frequency to get them violently swaying back and forth. Moreover, as mentioned previously, all musical instruments make their music by means of standing waves.

To create its sound, the physical structure of musical instruments is set into a standing wave condition. The connection with resonance can be seen with a trumpet, for example. The buzzing lips of the trumpet player create a sound wave by allowing a burst of air into the trumpet. This burst is largely reflected back when it reaches the end of the trumpet. If the trumpet player removes his lips, the sound wave naturally reflects back and forth between the beginning and the end of the trumpet, quickly dying out as it leaks from each end. However, if the player's lips stay in contact with the mouthpiece, the reflected burst of air can be reinforced with a new burst of air from the player's lips. The process continues, creating a standing wave of growing amplitude. Eventually the amplitude reaches the point where even the small portion of the standing wave that escapes the trumpet becomes audible. It is resonance because the trumpet player adds a "kick of air" at the precise frequency (and therefore also the same wavelength) of the already present standing sound wave.

INTRODUCTION TO MUSICAL INSTRUMENTS

IF YOU DON'T play a musical instrument, it's humbling to pick one up and try to create something that resembles a melody or tune. It gives you a true appreciation for the musician who seems to become one with his instrument. But you certainly don't have to be a musician to understand the physics of music or the physics of musical instruments. All musicians create music by making standing waves in their instrument. The guitar player makes standing waves in the strings of the guitar and the drummer does the same in the skin of the drumhead. The trumpet blower and flute player both create standing waves in the column of air within their instruments. Most kids have done the same thing, producing a tone as they blow over the top of a bottle. If you understand standing waves you can understand the physics of musical instruments. We'll investigate three classes of instruments:

- Chordophones (strings)
- Aerophones (open and closed pipes)
- Idiophones (vibrating rigid bars and pipes)

Most musical instruments will fit into these three categories. But to fully grasp the physics of the standing waves within musical instruments and corresponding music produced, an understanding of **wave impedance** is necessary.

Figure 2.4: Chordophones are musical instruments in which a standing wave is initially created in the strings of the instruments. Guitars, violins, and pianos fall into this category.

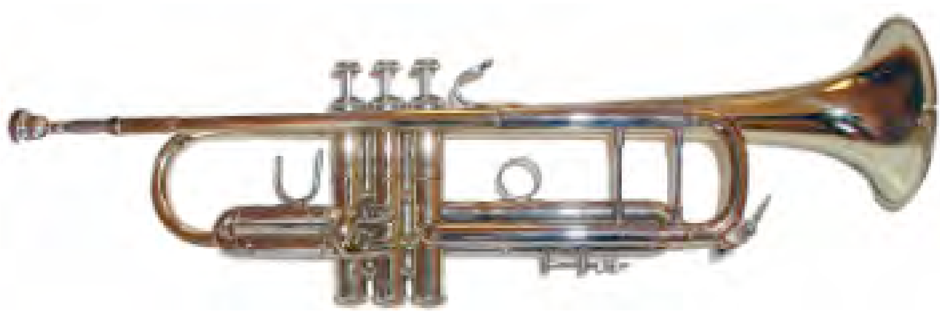


Figure 2.5: Aerophones are musical instruments in which a standing wave is initially created in the column of air within the instruments. Trumpets, flutes, and oboes fall into this category.

Figure 2.6: Idiophones are musical instruments in which a standing wave is initially created in the physical structure of the instruments. Xylophones, marimbas, and chimes fall into this category.

MUSICAL INSTRUMENTS AND WAVE IMPEDANCE

The more rigid a medium is the less effect the force of a wave will have on deforming the shape of the medium. Impedance is a measure of how much force must be applied to a medium by a wave in order to move the medium by a given amount. When it comes to standing waves in the body of a musical instrument, the most important aspect of impedance changes is that they always cause reflections. Whenever a wave encounters a change in impedance, some or most of it will be reflected. This is easy to see in the strings of a guitar. As a wave moves along the string and encounters the nut or bridge of the guitar, this new medium is much more rigid than the string and the change in impedance causes most of the wave to be reflected right back down the string (good thing, because the reflected wave is needed to create and sustain the standing wave condition). It's harder to see however when you consider the standing wave of air moving through the inside of the tuba. How is this wave reflected when it encounters the end of the tuba? The answer is that wave reflection occurs regardless of how big the impedance change is or whether the new impedance is greater or less. The percentage of reflection depends on how big the change in impedance is. The bigger the impedance change, the more reflection will occur. When the wave inside the tuba reaches the end, it is not as constricted – less rigid, so to speak. This slight change in impedance is enough to cause a significant portion of the wave to reflect back into the tuba and thus participate and influence the continued production of the standing wave. The part of the wave that is not reflected is important too. The transmitted portion of the wave is the part that constitutes the sound produced by the musical instrument. Figure 2.7 illustrates what happens when a wave encounters various changes in impedance in the medium through which it is moving.

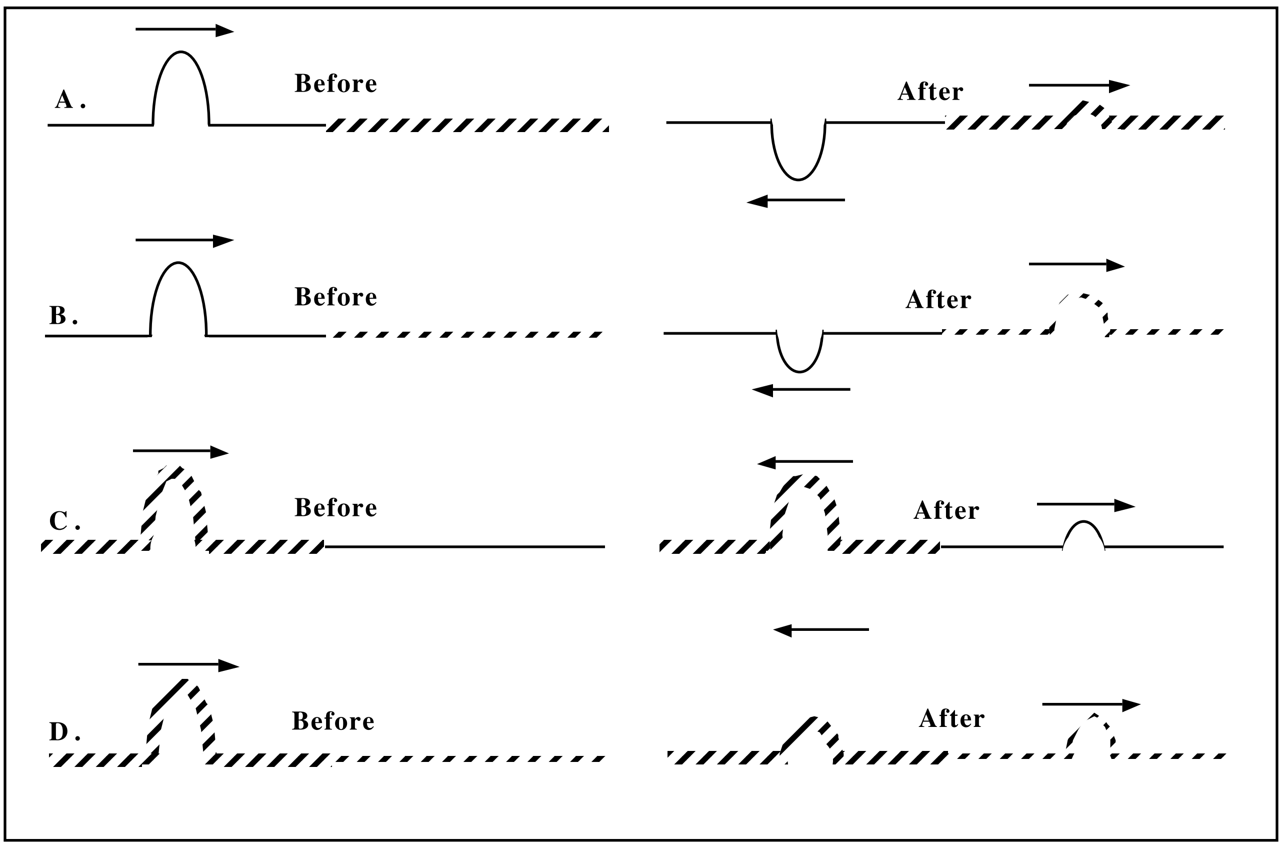


Figure 2.7: Wave pulse encountering medium with different impedance
A. Much greater impedance Inverted, large reflection.
B. Slightly greater impedance Inverted, small reflection.
C. Much smaller impedance Upright, large reflection.
D. Slightly smaller impedance Upright, small reflection.

"After silence that which comes nearest to expressing the inexpressible is music."
Aldous Huxley

CHAPTER 3
MODES, OVERTONES, AND HARMONICS

WHEN A DESIRED note (tone) is played on a musical instrument, there are always additional tones produced at the same time. To understand the rationale for the development of consonant musical scales, we must first discuss the different **modes** of vibration produced by a musical instrument when it is being played. It doesn't matter whether it is the plucked string of a guitar, or the thumped key on a piano, or the blown note on a flute, the sound produced by any musical instrument is far more complex than the single frequency of the desired note. The frequency of the desired note is known as the **fundamental frequency**, which is caused by the **first mode of vibration**, but many higher modes of vibration always naturally occur simultaneously.

Higher modes are simply alternate, higher frequency vibrations of the musical instrument medium. The frequencies of these higher modes are known as **overtones**. So the second mode produces the first overtone, the third mode produces the second overtone, and so on. In percussion instruments, (like xylophones and marimbas) the overtones are not related to the fundamental frequency in a simple way, but in other instruments (like stringed and wind instruments) the overtones are related to the fundamental frequency "harmonically."

When a musical instrument's overtones are harmonic, there is a very simple relationship between them and the fundamental frequency. **Harmonics** are overtones that happen to be simple integer multiples of the fundamental frequency. So, for example, if a string is plucked and it produces a frequency of 110 Hz, multiples of that 110 Hz will also occur at the same time: 220 Hz, 330 Hz, 440 Hz, etc will all be present, although not all with the same intensity. A musical instrument's fundamental frequency and all of its overtones combine to produce that instrument's **sound spectrum** or **power spectrum**. Figure 3.1 shows the sound spectrum from a flute playing the note G_4. The vertical line at the first peak indicates its frequency is just below 400 Hz. In the musical scale used for this flute, G_4 has a frequency of 392 Hz. Thus, this first peak is the desired G_4 pitch. The second and third peaks are also identified with vertical lines and have frequencies of about 780 Hz and 1,170 Hz (approximately double and triple the lowest frequency of 392 Hz). This lowest frequency occurring when the G_4 note is played on the flute is caused by the first mode of vibration. It is the fundamental frequency. The next two peaks are the simultaneously present frequencies caused by the second and third modes of vibration. That makes them the first two overtones. Since these two frequencies are integer multiples of the lowest frequency, they are harmonic overtones.

When the frequencies of the overtones are harmonic the fundamental frequency and all the overtones can be classified as some order of harmonic. The fundamental frequency becomes the **first harmonic**, the first overtone becomes the **second harmonic**, the second overtone becomes the **third harmonic**, and so on. (This is usually confusing for most people at first. For a summary, see Table 3.1). Looking at the Figure 3.1, it is easy to see that there are several other peaks that appear to be integer multiples of the fundamental frequency. These are all indeed higher harmonics. For this flute, the third harmonic is just about as prominent as the fundamental. The second harmonic is a bit less than either the first or the third. The fourth and higher harmonics are all less prominent and generally follow a pattern of less prominence for higher harmonics (with the exception of the eighth and ninth harmonics). However, only the flute has this particular "spectrum of sound." Other instruments playing G_4 would have overtones present in different portions. You could say that the sound spectrum of an instrument is its "acoustical fingerprint."

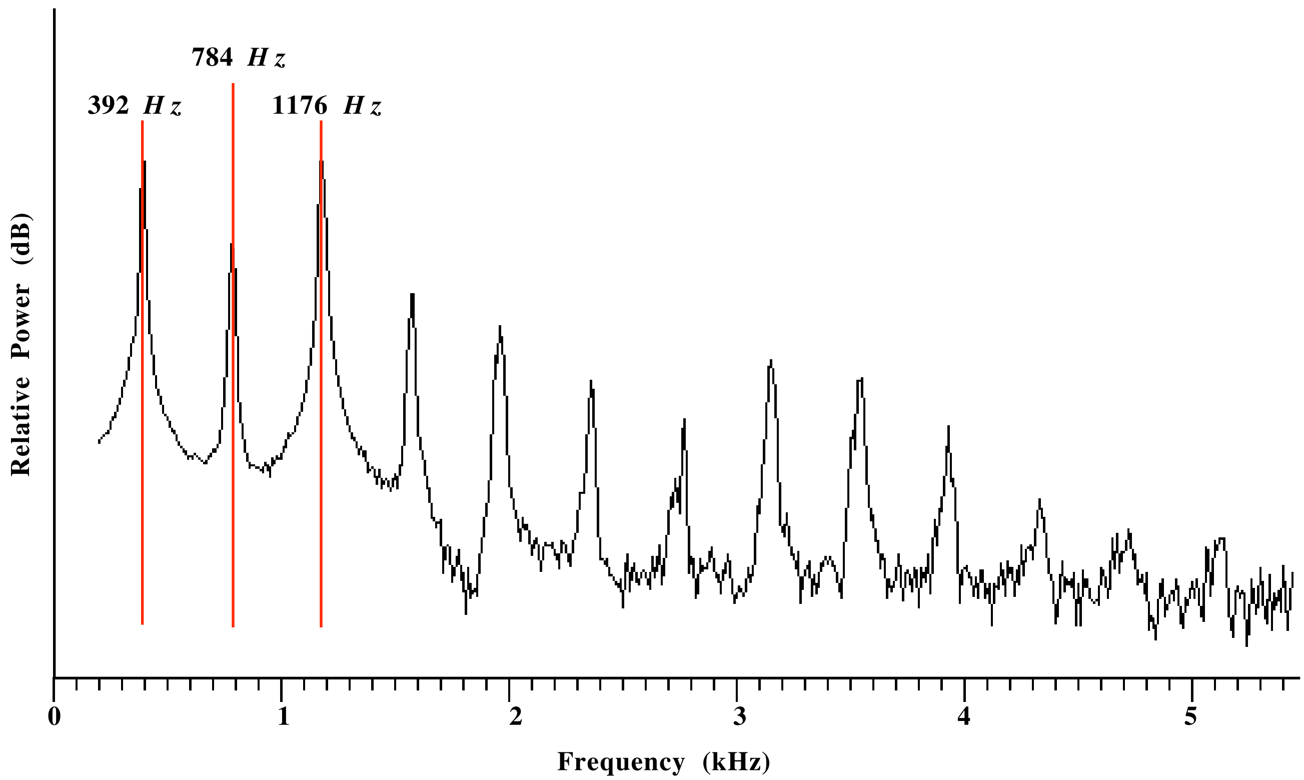


Figure 3.1: The "sound spectrum" of a flute shows the frequencies present when the G_4 note is played. The first designated peak is the desired frequency of 392 *Hz* (the "fundamental frequency"). The next two designated peaks are the first and second "overtones." Since these and all higher overtones are integer multiples of the fundamental frequency, they are "harmonic." (Used with permission. This and other sound spectra can be found at http://www.phys.unsw.edu.au/music/flute/modernB/G4.html)

Mode of vibration	Frequency name (for any type of overtone)	Frequency name (for harmonic overtones)
First	Fundamental	First harmonic
Second	First overtone	Second harmonic
Third	Second overtone	Third harmonic
Fourth	Third overtone	Fourth harmonic

Table 3.1: Names given to the frequencies of different modes of vibration. The overtones are "harmonic" if they are integer multiples of the fundamental frequency.

TIMBRE, THE QUALITY OF SOUND

If your eyes were closed, it would still be easy to distinguish between a flute and a piano, even if both were playing the note G_4. The difference in intensities of the various overtones produced gives each instrument a characteristic **sound quality** or **timbre** ("tam-brrr"), even when they play the same note. This ability to distinguish is true even between musical instruments that are quite similar, like the clarinet and an oboe (both wind instruments using physical reeds). The contribution of total sound arising from the overtones varies from instrument to instrument, from note to note on the same instrument and even on the same note (if the player produces that note differently by blowing a bit harder, for example). In some cases, the power due to the overtones is less prominent and the timbre has a very pure sound to it, like in the flute or violin. In other instruments, like the bassoon and the bagpipes, the overtones contribute much more significantly to the power spectrum, giving the timbre a more complex sound.

A CLOSER LOOK AT THE PRODUCTION OF OVERTONES

To begin to understand the reasons for the existence of overtones, consider the guitar and how it can be played. A guitar string is bound at both ends. If it vibrates in a standing wave condition, those bound ends will necessarily have to be nodes. When plucked, the string can vibrate in any standing wave condition in which the ends of the string are nodes and the length of the string is some integer number of half wavelengths of the mode's wavelength (see Figure 3.2). To understand how the intensities of these modes might vary, consider plucking the string at its center. Plucking it at its center will favor an antinode at that point on the string. But all the odd modes of the vibrating string have antinodes at the center of the string, so all these modes will be stimulated. However, since the even modes all have nodes at the center of the string, these will generally be weak or absent when the string is plucked at the center, forcing this point to be moving (see Figure 3.2). Therefore, you would expect the guitar's sound quality to change as its strings are plucked at different locations.

An additional (but more subtle) explanation for the existence and intensity of overtones has to do with how closely the waveform produced by the musical instrument compares to a simple sine wave. We've already discussed what happens when a flexible physical system is forced from its position of greatest stability (like when a pendulum is moved from its rest position or when the tine of a garden rake is pulled back and then released). A force will occur that attempts to restore the system to its former state. In the case of the pendulum, gravity directs the pendulum back to its rest position. Even though the force disappears when the pendulum reaches this rest position, its momentum causes it to overshoot. Now a new force, acting in the opposite direction, again attempts to direct the pendulum back to its rest position. Again, it will overshoot. If it weren't for small amounts of friction and some air resistance, this back and forth motion would continue forever. In its simplest form, the amplitude vs. time graph of this motion would be a sine wave (see Figure 3.3). Any motion that produces this type of graph is known as **simple harmonic motion**.

Not all oscillatory motions are simple harmonic motion though. In the case of a musical instrument, it's not generally possible to cause a physical component of the instrument (like a string or reed) to vibrate as simply as true simple harmonic motion. Instead, the oscillatory motion will be a more complicated repeating waveform (Figure 3.4). Here is the "big idea." This more complicated waveform can always be created by adding in various intensities of waves that are integer multiples of the fundamental frequency. Consider a hypothetical thumb piano tine vibrating at 523 Hz. The tine physically cannot produce true simple harmonic motion when it is plucked. However, it is possible to combine a 1,046 Hz waveform and a 1,569 Hz waveform with the fundamental 523 Hz waveform, to create the waveform produced by the actual vibrating tine. (The 1,046 Hz and 1,569 Hz frequencies are integer multiples of the fundamental 523 Hz frequency and do not necessarily have the same intensity as the fundamental frequency). The total sound produced by the tine would then have the fundamental frequency of 523 Hz as well as its first two overtones.

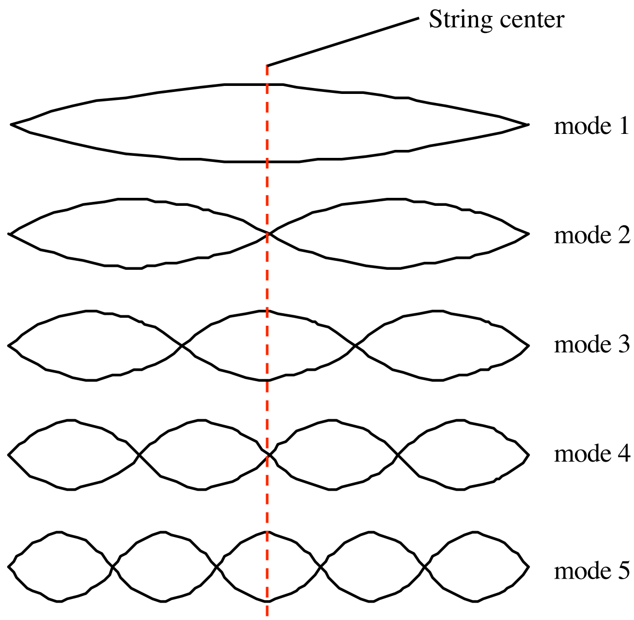


Figure 3.2: The first five modes of a vibrating string. Each of these (and all other higher modes) meets the following criteria: The ends of the string are nodes and the length of the string is some integer number of half wavelengths of the mode's wavelength. The red dashed line indicates the center of the string. Note that if the center of the string is plucked, forcing this spot to move, modes 2 and 4 (and all other even modes) will be eliminated since they require a node at this point. However, modes 1, 3, and 5 (and all other odd modes) will all be stimulated since they have antinodes at the center of the string.

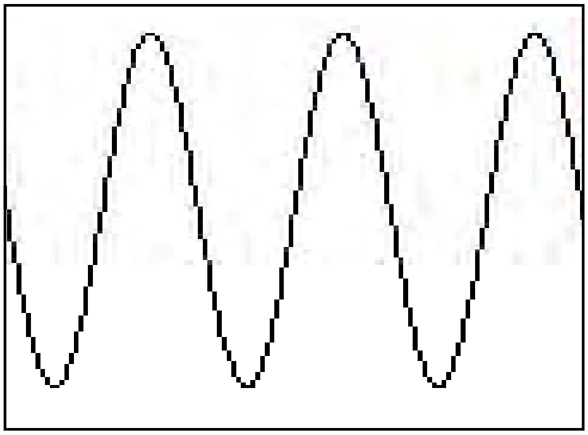


Figure 3.3: The sine wave pictured here is the displacement vs. time graph of the motion of a physical system undergoing simple harmonic motion.

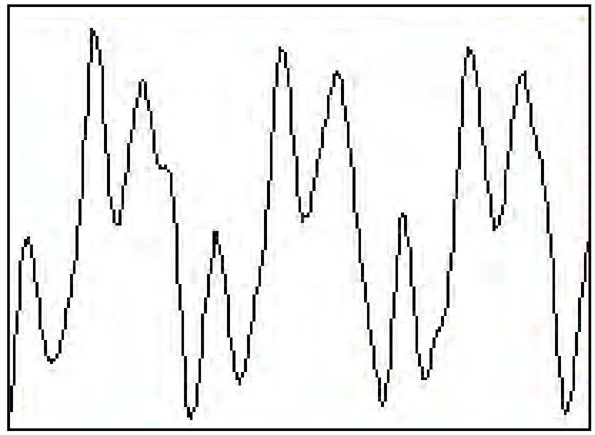


Figure 3.4: The complicated waveform shown here is typical of that produced by a musical instrument. This waveform can be produced by combining the waveform of the fundamental frequency with waveforms having frequencies that are integer multiples of the fundamental frequency.

The remainder of this section provides visualization for the timbre of a variety of musical instruments. Each figure provides two graphs for a particular instrument. The first graph is a histogram of the power spectrum for that musical instrument. The first bar on the left is the power of the fundamental frequency, followed by the overtones. Be careful when comparing the relative strength of the overtones to the fundamental and to each other. The scale of the vertical axis is logarithmic. The top of the graph represents 100% of the acoustic power of the instrument and the bottom of the graph represents 80 dB lower than full power (10^{-8} less power). The second graph is a superposition of the fundamental wave together with all the overtone waves. Three cycles are shown in each case. Notice that where graphs for the same instrument are shown that this acoustical fingerprint varies somewhat from note to note – the bassoon almost sounds like a different instrument when it goes from a very low note to a very high note.

You can click on the name of the instrument in the caption to hear the sound that produced both graphs.

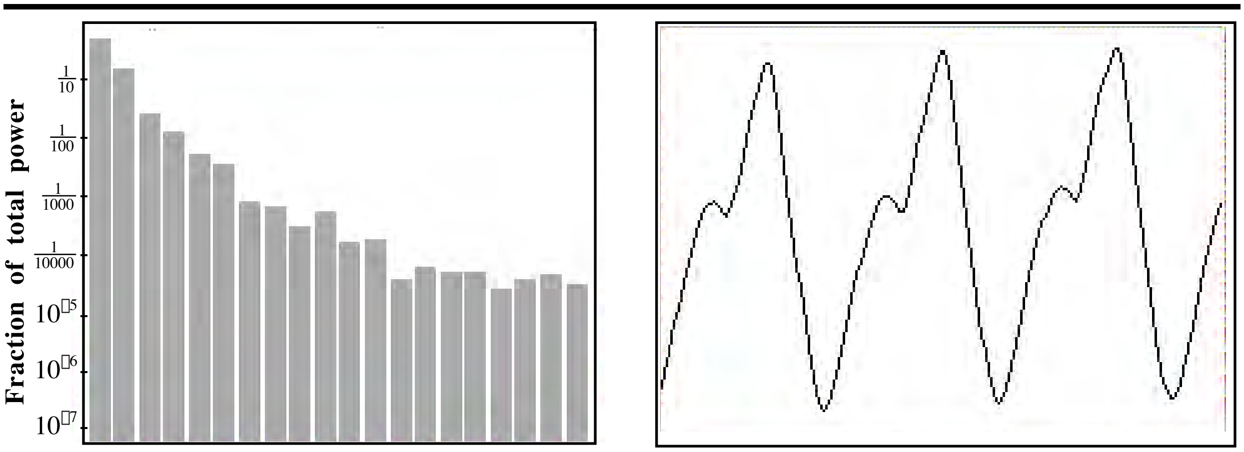


Figure 3.5: E_5 played on the VIOLIN. Note the dominance of the fundamental frequency. The most powerful overtone is the 1st, but only slightly more than 10% of the total power. The higher overtones produce much less power. Of the higher overtones, only the 2nd overtone produces more than 1% of the total power. The result is a very simple waveform and the characteristically pure sound associated with the violin.

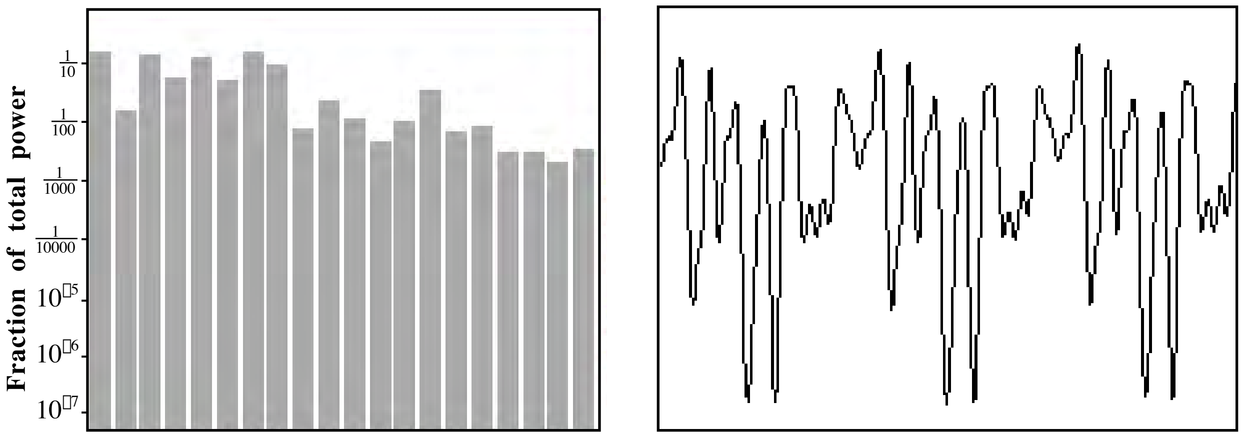


Figure 3.6: F_3 played on the CLARINET. Note that although all harmonics are present, the 1st, 3rd, 5th, and 7th harmonics strongly dominate. They are very nearly equal to each other in power. This strong presence of the lower odd harmonics gives evidence of the closed pipe nature of the clarinet. The presence of these four harmonics in equal proportion (as well as the relatively strong presence of the 8th harmonic) creates a very complex waveform and the clarinet's characteristically "woody" sound

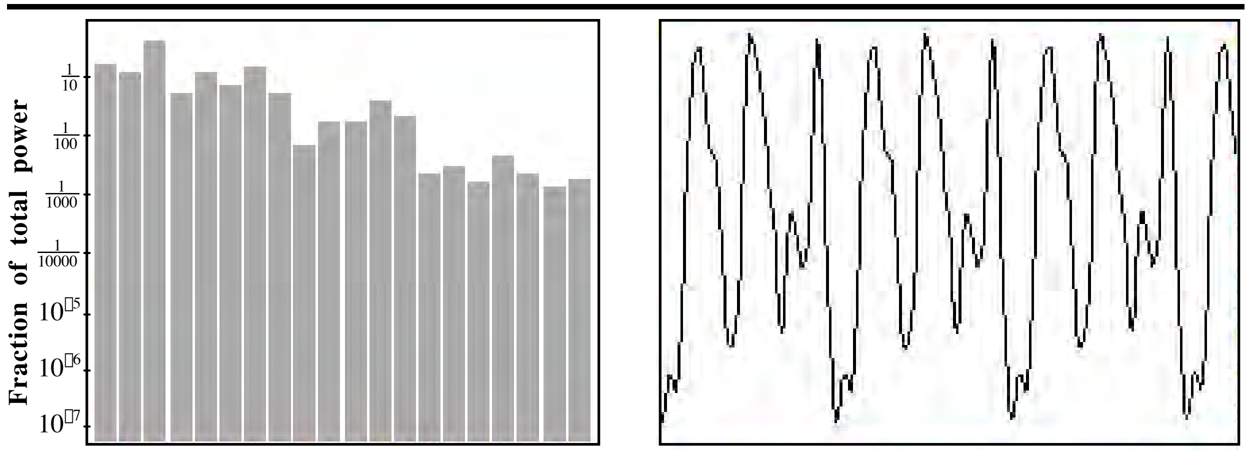


Figure 3.7: B_4 played on the BAGPIPE. There is a strong prominence of odd harmonics (through the 7th). This is a hint about the closed pipe nature of the bagpipe's sound production. The 3rd harmonic (2nd overtone) is especially strong, exceeding all other mode frequencies in power. Not only is the third harmonic more powerful than the fundamental frequency, it appears to have more power than all the other modes combined. This gives the bagpipe its characteristic harshness.

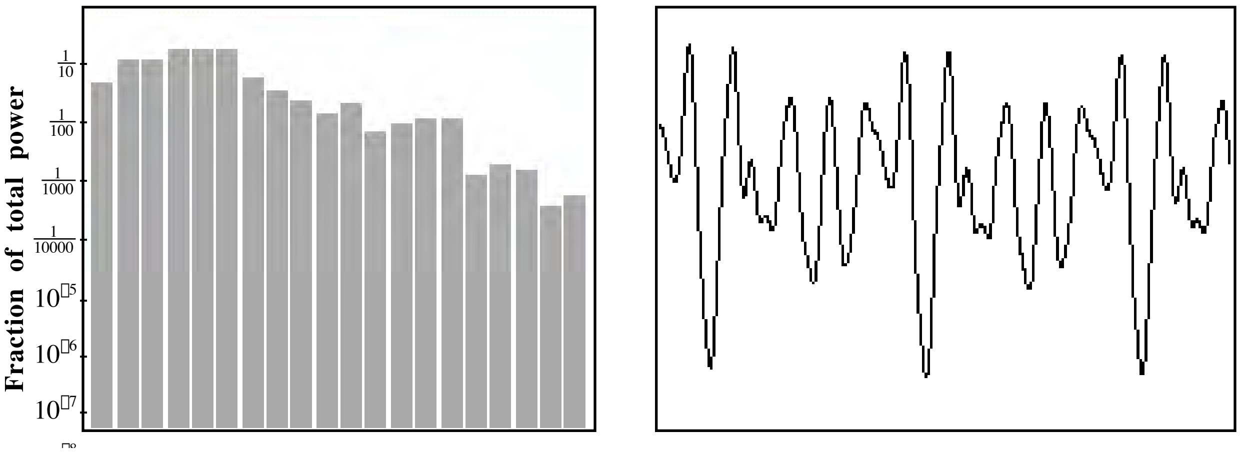


Figure 3.8: F_2 played on the BASSOON. Note that the fundamental frequency produces far less than 10% of the total power of this low note. It's also interesting to note that the first five overtones not only produce more power than the fundamental, but they each individually produce more than 10% of the total power of this note. This weak fundamental combined with the dominance of the first five harmonics creates a very complex waveform and the foghorn-like sound of the bassoon when it produces this note.

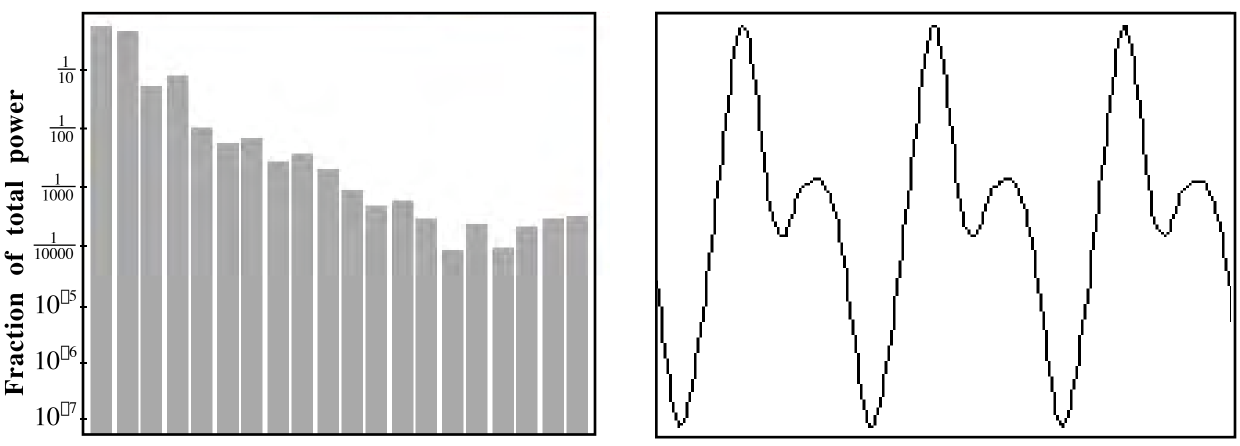


Figure 3.9: B_4 played on the BASSOON. The power spectrum is much different for this high note played on the same bassoon. Over two octaves higher than the note producing the power spectrum in Figure 3.8, the first two harmonics dominate, producing almost all the power. The third and fourth harmonics produce far less than 10% of the total power and all the remaining higher harmonics produce less than 1% combined. The result is a simple waveform with an almost flute-like sound.

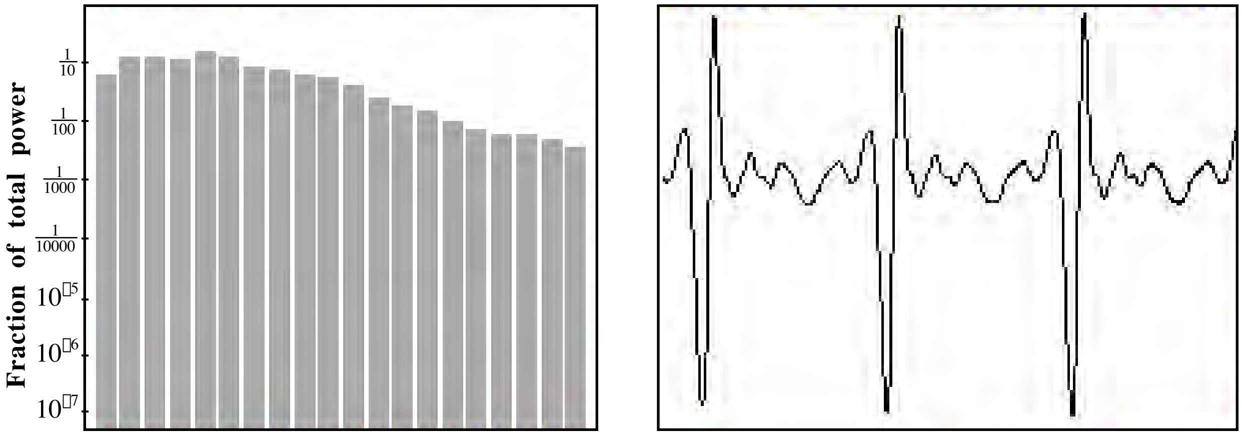


Figure 3.10: B^b_3 played on the TRUMPET. Like the F_2 note on the bassoon, the fundamental frequency here produces less than 10% of the total power of this low note. Also like the bassoon's F_2, the first five overtones here not only produce more power than the fundamental, but they each individually produce more than 10% of the total power of this note. There are significant similarities between the two power spectra, but the seeming subtle differences on the power spectra graph become far more obvious when either comparing the two waveform graphs or listening to the notes.

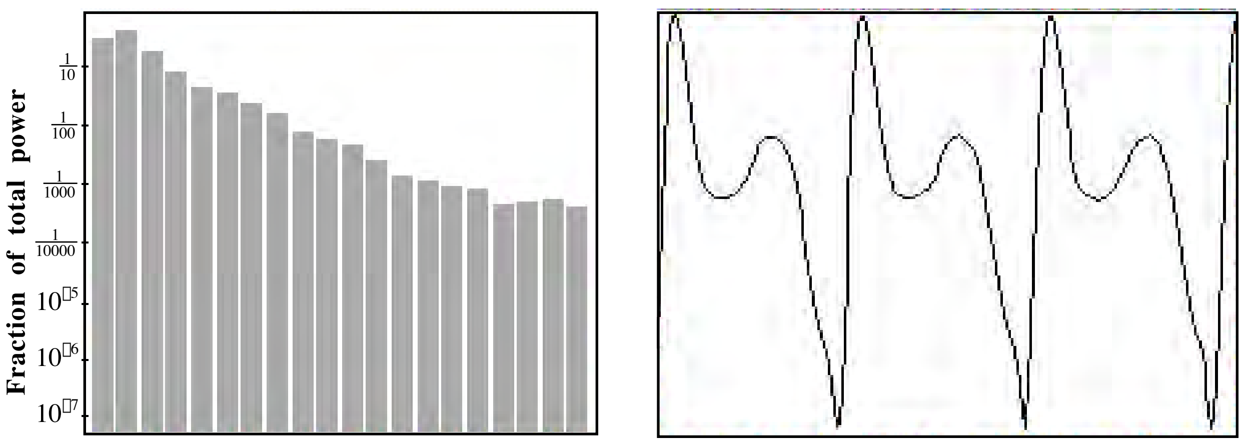


Figure 3.11: D_5 played on the TRUMPET. The fundamental frequency and the first two overtones dominate the power spectrum, with all three contributing over 10% of the total power. The next five harmonics all contribute above 1% of the total power. However, even with the much greater prominence of the 3rd through 7th overtones, when compared to the bassoon's B_4 note, the waveform graphs of the trumpet D_5 and the bassoon B_4 are surprisingly prominent.

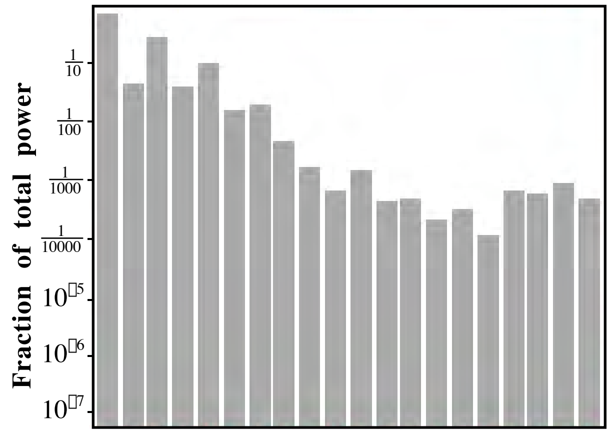
ACTIVITY

INTERPRETING MUSICAL INSTRUMENT POWER SPECTRA

In this activity, you will view the power spectrum graphs for a pure tone as well as three musical instruments: a flute (open end), a panpipe (closed end), and a saxophone. The power spectra are presented randomly below. For each power spectrum you will be asked to:

- Match the power spectrum to one of four waveform graphs.
- Make a general statement about the timbre of the sound (pure, complex, harsh, clarinet-like, etc.).
- Match the power spectrum to the correct musical instrument or to the pure tone.

1. a. Which of the waveforms shown at the end of this activity corresponds to this power spectrum? Explain.



b. Describe the timbre of this sound. Explain.

c. Predict which, if any, of the musical instruments listed above produced this power spectrum. Explain.

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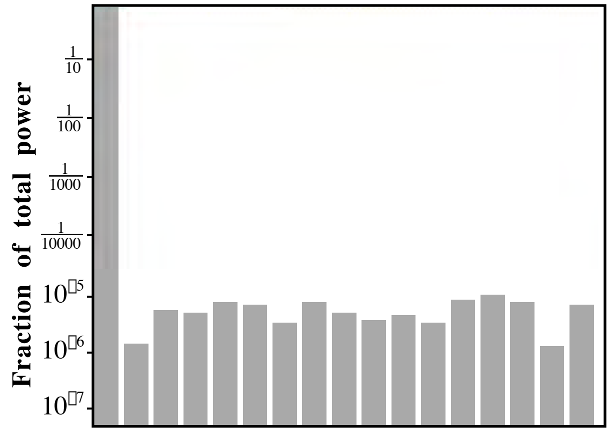
2. a. Which of the waveforms shown at the end of this activity corresponds to this power spectrum? Explain.



b. Describe the timbre of this sound. Explain.

c. Predict which, if any, of the musical instruments listed above produced this power spectrum. Explain.

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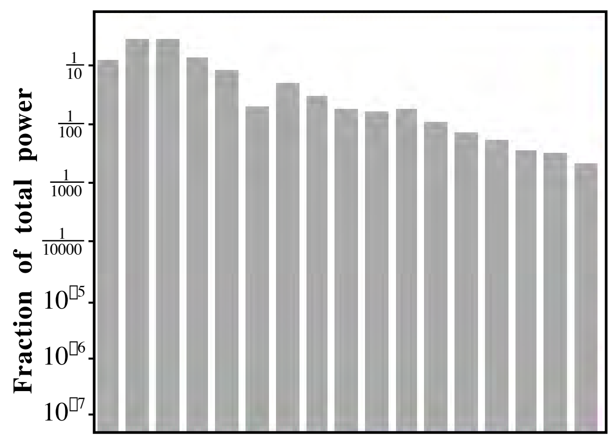
3. a. Which of the waveforms shown at the end of this activity corresponds to this power spectrum? Explain.



b. Describe the timbre of this sound. Explain.

c. Predict which, if any, of the musical instruments listed above produced this power spectrum. Explain.

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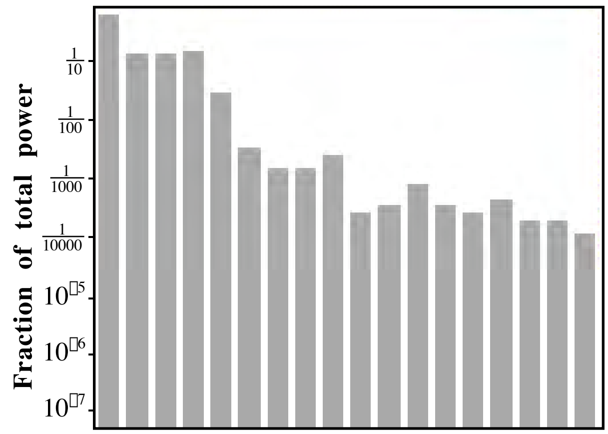
4. a. Which of the waveforms shown at the end of this activity corresponds to this power spectrum? Explain.



b. Describe the timbre of this sound. Explain.

c. Predict which, if any, of the musical instruments listed above produced this power spectrum. Explain.

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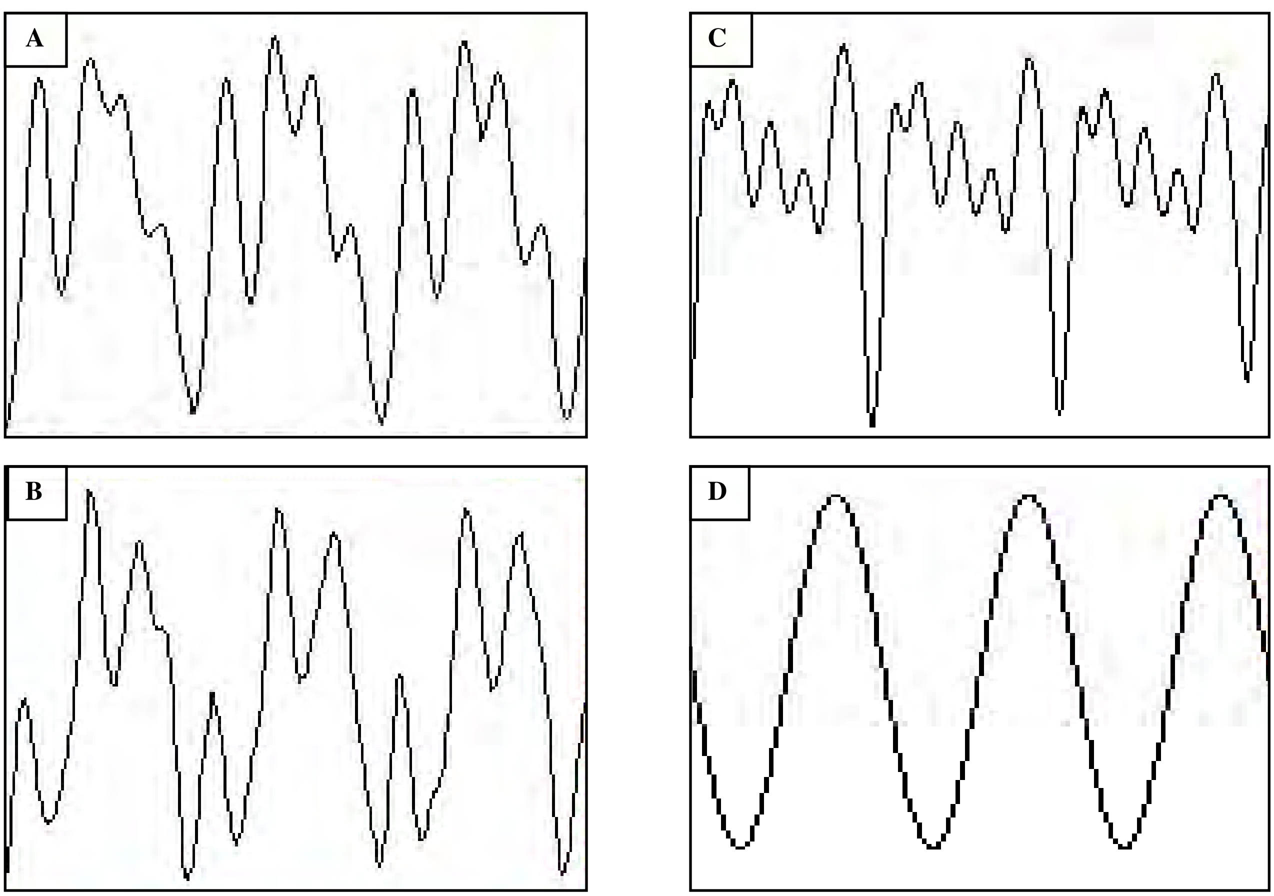


BEGINNING TO THINK ABOUT MUSICAL SCALES

NOW THAT YOU know a bit about waves and even more so about the sound waves that emerge from musical instruments, you're ready to start thinking about musical scales. In some cultures, the music is made primarily with percussion instruments and certain common frequencies are far less important than rhythm. But in most cultures, musical instruments that produce sustained frequencies are more prominent than percussion instruments. And, for these instruments to be played simultaneously, certain agreed upon frequencies must be adopted. How did musicians come up with widely agreed upon common frequencies used in musical scales? How would you do it? Take a moment to consider that question before you move on. What aspects of the chosen frequencies would be important in the development of your scale?

Use the space below to make some proposals for what you believe would be important as you begin to build a musical scale.

Now show your proposals to a neighbor and have that person provide feedback concerning your ideas. Summarize this feedback below.

BEATS

ONE WAY TO tune a guitar is to compare the frequencies of various combinations of pairs of the guitar strings. It's easy to tell whether two strings are different in frequency by as little as a fraction of one hertz. And, it's not necessary to have a great ear for recognizing a particular frquency. This type of tuning is dependent on the type of wave intereference that produces **beats**. In the same way as two slinky waves will interfere with each other, either reinforcing each other in **constructive interference** or subtracting from each other in **destructive interference**, sound waves moving through the air will do the same. With sound waves from two sources (like two guitar strings), constructive interference would correspond to a sound louder than the two individually and destructive interference would correspond to a quieter sound than either, or perhaps … absolute silence (if the amplitudes of the two were the same). Figure 3.12 attempts to illustrate this. Two tuning forks with slightly different frequencies are sounded at the same time in the same area as a listening ear. The closely packed black dots in front of the tuning forks represent compressions in the air caused by the vibrations of the forks. These compressions have higher than average air pressure. The loosely packed hollow dots in front of the tuning forks represent "anti-compressions" or rarefactions in the air, also caused by the vibrations of the forks. These rarefactions have lower than average air pressure. When the top tuning fork has produced 17 compressons, the bottom has produced 15. If the time increment for this to occur were half a second, a person listening to one or the other would hear a frequency of 34 Hz from the top tuning fork and 30 Hz from the bottom tuning fork. But listening to the sound from both tuning forks at the same time, the person would hear the combination of the two sound waves, that is, their interference. Notice that there are regions in space where compressions from both tuning forks combine to produce an especially tight compression, representing a sound amplitude maximum – it's especially loud there. There are other locations where a compression from one tuning fork is interfering with a rarefaction from the other tuning fork. Here the compression and the rarefaction combine to produce normal air pressure – no sound at all. At locations in between these two extremes in interference, the sound amplitude is either growing louder or quieter. You can see in the bottom of the diagram that there is a rhythm of sound intensity from loud to silent to loud, over and over. This is the phenomenon of *beats*. In the half second that the diagram portrays there are two full cycles of this beating, giving a **beat frequency** of 4 Hz. The ear would perceive the average frequency of these two tuning forks, 32 Hz, getting louder and quieter four times per second (**note that this beat frequency is the difference in the frequency of the two tuning forks**).

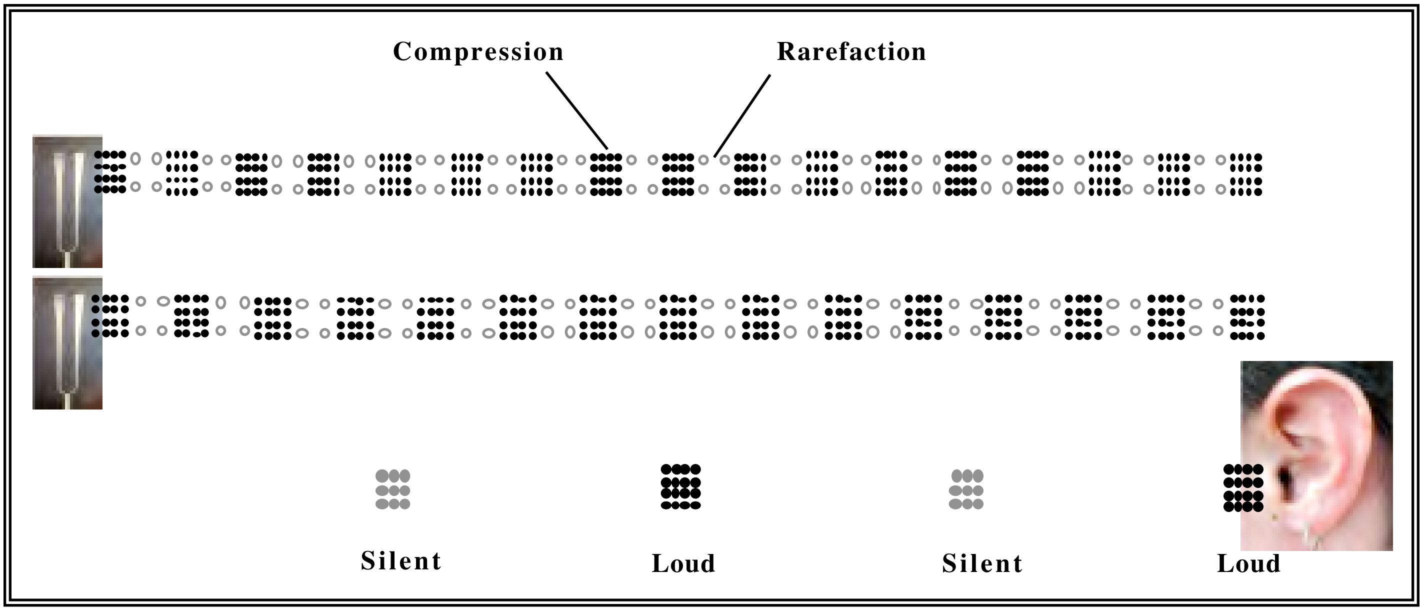


Figure 3.12: Beats. The two tuning forks have slightly different frequencies. This causes the sound waves produced by each one to interfere both constructively and destructively at various points. The constructive interference causes a rise in the intensity of the sound and the destructive interference causes silence. This pattern is repeated over and over causing the phenomenon of beats.

The beat frequency decreases as the two frequencies causing the beats get closer to each other. Finally, the beat frequency disappears when the two frequencies are identical. This is why it is so easy to tune two guitar strings to the same frequency. You simply strum both strings together and then tune one until no beat frequency is heard. At that point, the two frequencies are the same. Click on the following links to hear demonstrations of beats caused by various combinations of audio frequencies:

300 Hz and 301 Hz
300 Hz and 302 Hz
300 Hz and 305 Hz
300 Hz and 310 Hz

Beats Movie

CRITICAL BANDS AND DISSONANCE

In addition to considering the issue of beats when choosing frequencies for a musical scale, there is also the issue of **critical bands**. When sound enters the ear, it ultimately causes vibrations on the basilar membrane within the inner ear. Different frequencies of sound cause different regions of the basilar membrane and its fine hairs to vibrate. This is how the brain discriminates between various frequencies. However, if two frequencies are close together, there is an overlap of response on the basilar membrane – a large fraction of total hairs set into vibration are caused by both frequencies (see figure 3.13).

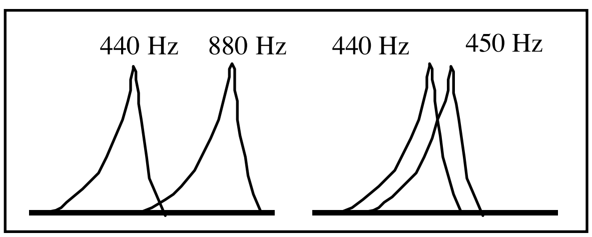


Figure 3.13: The closer two frequencies are to each other, the more overlap there will be in the response of the basilar membrane within the inner ear.

When the frequencies are nearly the same, they can't be distinguished as separate frequencies. Instead an average frequency is heard, as well as the beats discussed above. If the two frequencies were 440 Hz and 450 Hz, you would hear 445 Hz beating at 10 times per second. If the lower frequency were kept at 440 Hz and the higher one were raised slowly, there would come a point where the two frequencies were still indistinguishable, but the beat frequency would be too high to make out. There would just be a roughness to the total sound. This dissonance would continue until finally the higher frequency would become distinguishable from the lower. At this point, further raising the higher frequency would cause less and less dissonance. When two frequencies are close enough to cause the beating and roughness described above, they are said to be within a critical band on the basilar membrane. For much of the audible range, the critical band around some "central frequency" will be stimulated by frequencies within about 15% of that central frequency.

SUMMARY

When two tones with similar frequencies, f_1 and f_2, are sounded in the same space, their interference will cause beats, the increase and decrease of perceived sound intensity – a throbbing sensation. The perceived frequency is the average of the two frequencies:

$$f_{perceived} = \frac{f_1 + f_2}{2}$$

The beat frequency (rate of the throbbing) is the difference of the two frequencies:

$$f_{beat} = |f_1 - f_2|$$

Example

If two people stood near each other and whistled, one with a frequency of 204 Hz and the other with a frequency of 214 Hz, what would people near them hear?

The observers would hear a pitch that was the average of the two frequencies, but beating at a frequency equal to the difference of the two frequencies:

Given: $f_1 = 204\ Hz$
$f_2 = 214\ Hz$

Find: The perceived and beat frequencies

$$f_{perceived} = \frac{f_1 + f_2}{2} = \frac{204Hz + 214Hz}{2} = 209Hz$$

The observers would hear the frequency of 209 Hz getting louder and softer 10 times per second.

When considering the frequencies to use for a musical scale, critical bands should be considered. Two frequencies that stimulate areas within the same critical band on the basilar membrane will produce either noticeable beats (if they are similar enough to each other) or other dissonance undesireable in music.

"The notes I handle no better than many pianists. But the pauses between the notes – ah, that is where the art resides."
– Arthur Schnabel

CHAPTER 4
MUSICAL SCALES

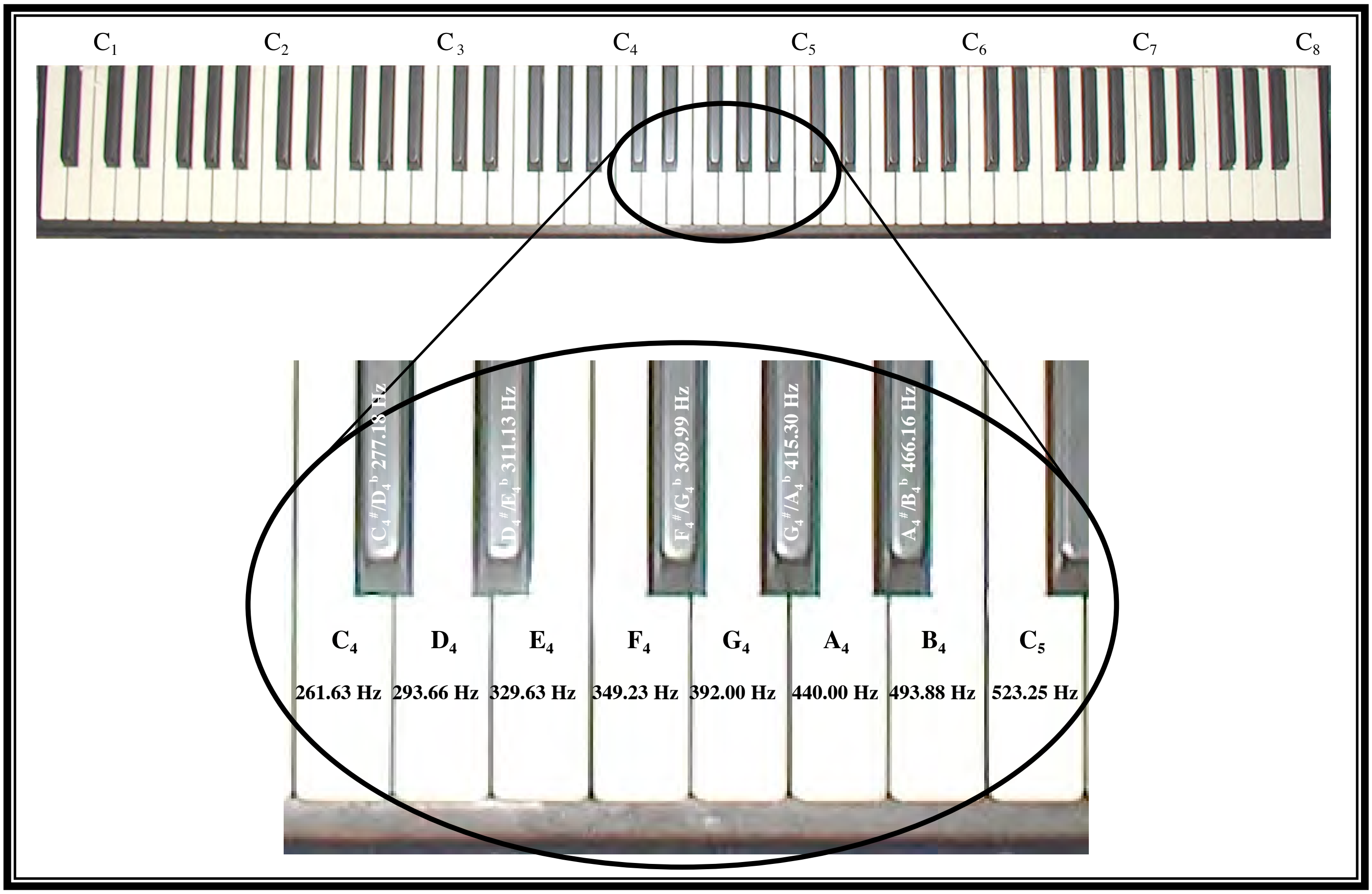


Figure 4.1: The repeating nature of the musical scale is illustrated on a piano keyboard.

THE PHOTOGRAPH OF the piano keyboard in Figure 4.1 shows several of the white keys with the letter *C* above them. The keys are all equally spaced and striking all keys in succession to the right from any one *C* key to the next *C* key would play the familiar sound of "*Do – Re – Me – Fa – So – La – Ti – Do.*" That means every *C* key sounds like "*Doe*" in the familiar singing of the musical scale.

The reason for the equally spaced positioning of the *C*'s is because there is a repetition that takes place as you move across the piano keyboard – all the *C* keys sounds alike, like "*Do*." And every key immediately to the right of a *C* key sounds like "*Re*." So if you want to understand the musical scale, you really only need to look at the relationship between the keys in one of these groupings from *C* to *C* (or from any key until the next repeated key).

The particular frequencies chosen for the musical scale are not random. Many keys sound particularly good when played together at the same time. To help show why certain pitches or tones sound good together and why they are chosen to be in the scale, one grouping of notes has been magnified from the photograph of the piano keyboard. The seven white keys (from C_4 to B_4 represent the **major diatonic scale**). The black keys are intermediate tones. For example, the black key in between D_4 and E_4 is higher frequency (sharper) than D_4 and lower frequency (flatter) than E_4. This note is therefore known synonymously as either "*D_4 sharp*" ($D_4^{\#}$) or "*E_4 flat*" (E_4^{b}). Including these five sharps or flats with the other seven notes gives the full **chromatic scale**. The frequency of the sound produced by each of the keys in this chromatic scale (as well as for the first white key in the next range) is shown on that key. It will help to refer back to this magnified portion of the keyboard as you consider the development of the musical scale.

CONSONANCE AND SMALL INTEGER FREQUENCY RATIOS

"All art constantly aspires towards the condition of music."
– Walter Pater

The opposite of dissonance is **consonance** – pleasant sounding combinations of frequencies. Earlier the simultaneous sounding of a 430 Hz tuning fork with a 440 Hz tuning fork was discussed. If the 430 Hz tuning fork were replaced with an 880 Hz tuning fork, you would hear excellent consonance. This especially pleasant sounding combination comes from the fact that every crest of the sound wave produced by the 440 Hz tuning fork would be in step with every other crest of the sound wave produced by the 880 Hz tuning fork. So doubling the frequency of one tone always produces a second tone that sounds good when played with the first. The Greeks knew the interval between these two frequencies as a **diapson**. 440 Hz and 880 Hz sound so good together, in fact, that they sound … the same. As the frequency of the 880 Hz tone is increased to 1760 Hz, it sounds the same as when the frequency of the 440 Hz tone is increased to 880 Hz. This feature has led widely different cultures to historically use a one arbitrary frequency and another frequency, exactly one diapson higher, as the first and last notes in the musical scale. Diapson means literally "through all." If you sing the song, *Somewhere Over the Rainbow*, the syllables Some-where differ in frequency by one diapson. As mentioned above, frequencies separated by one diapson not only sound good together, but they *sound like* each other. So an adult and a child or a man and a woman can sing the same song together simply by singing in different diapsons. And they'll do this naturally, without even thinking about it.

The next step in the development of the musical scale is to decide how many different tones to incorporate and how far apart in frequency they should be. If a certain frequency and another one twice as high act as the first and last notes for the scale, then other notes can be added throughout the range. Two reasonable constraints are that the frequencies chosen will be fairly evenly spaced and that they will sound good when played together.

Constraints for Choosing Frequencies for a Musical Scale

- Even Spacing
- Consonance when Played Together

The Greek mathematician, Pythagoras, experimented plucking strings with the same tension, but different lengths. This is easy to do with a monochord supported by a movable bridge (see figure 4.2). He noticed that when two strings (one twice as long as the other) were plucked at the same time, they sounded good together. Of course he didn't know anything about the difference in the frequencies between the two, but he was intrigued by the simplicity of the 2:1 ratio of the lengths of the two strings. When he tried other simple ratios of string lengths (2:1, 3:2, and 4:3) he found that he also got good consonance. We now know that if the tension in a string is kept constant and its length is changed, the frequency of sound produced when the

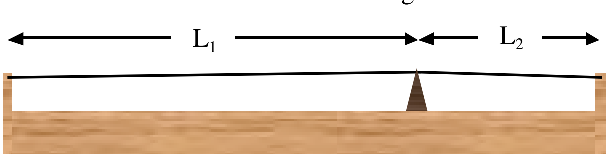


Figure 4.2: A *monochord*. The movable bridge turns its one vibrating string into two vibrating strings with different lengths but the same tension.

string is plucked will be inversely proportional to its change in length – twice the length gives half the frequency and one-third the length gives three times the frequency. So a frequency of 1,000 Hz sounds good when sounded with 2,000 Hz $\left(\frac{2{,}000}{1{,}000} = \frac{2}{1}\right)$, 1,500 Hz $\left(\frac{1{,}500}{1{,}000} = \frac{3}{2}\right)$, and 1,333 Hz $\left(\frac{1{,}333}{1{,}000} \approx \frac{4}{3}\right)$. One of the theories given for this consonance is that the frequencies will neither be similar enough to cause beats nor be within the same critical band.

We're now in a position to pull together the ideas of modes, overtones, and harmonics to further explain the consonance of tones whose frequencies are ratios of small integers. Recall that one theory for consonance is that simultaneously sounded frequencies will neither be similar enough to cause beats nor be within the same critical band. A second theory for this consonance is that many of the overtones of these two frequencies will coincide and most of the ones that don't will neither cause beats nor be within the same critical band.

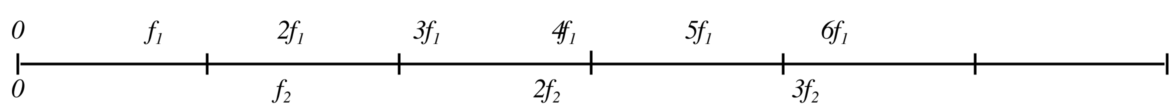
Theories for Consonance Between Two Frequencies with Small Integer Ratios

- The frequencies will neither be similar enough to cause beats nor be within the same critical band.
- Many of the overtones of these two frequencies will coincide and most of the ones that don't will neither cause beats nor be within the same critical band.

The following two examples illustrate these theories for the consonance between two frequencies with small integer ratios.

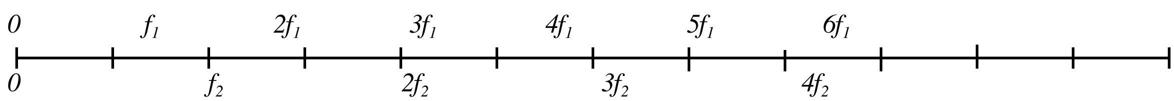
Think of a musical instrument that produces harmonic overtones (most instruments do). Let's consider two fundamental frequencies it can produce: f_1, = 100 Hz and another frequency, f_2, = 200 Hz. f_2 has a ratio with f_1 of 2:1. The table to the right and graph below show the harmonics for each frequency. Notice that all of the harmonics of f_2 are identical to a harmonic of f_1.

Frequency 1 (Hz)	Frequency 2 (Hz)
$f_1 = 100$	$f_2 = 200$
$2f_1 = 200$	$2f_2 = 400$
$3f_1 = 300$	$3f_2 = 600$
$4f_1 = 400$	
$5f_1 = 500$	
$6f_1 = 600$	



You can see similar circumstances between another combination of the instrument's fundamental frequencies: f_1 = 100 Hz and f_2, = 150 Hz. $\frac{f_2}{f_1} = \frac{150\,Hz}{100\,Hz} = \frac{3}{2}$, so f_2 has a ratio with f_1 of 3:2. The table to the right and graph below show the harmonics for each frequency. The match of harmonics is not quite as good, but the harmonics of f_2 that don't match those of f_1 are still different enough from the harmonics of f_1 that no beats are heard and they don't fall within the same critical band.

Frequency 1 (Hz)	**Frequency 2 (Hz)**
$f_1 = 100$	$f_2 = 150$
$2f_1 = 200$	$2f_2 = 300$
$3f_1 = 300$	$3f_2 = 450$
$4f_1 = 400$	$4f_2 = 600$
$5f_1 = 500$	
$6f_1 = 600$	



ACTIVITY

CONSONANCE

Do the same types of graphs as shown on the previous page, but for pairs of frequencies that are in ratios of other small integers: $\frac{f_2}{f_1} = \frac{4}{3}$ *and* $\frac{f_2}{f_1} = \frac{5}{4}$. Make each graph long enough so that there are at least eight harmonics showing for f_2.

Compare the level of consonance that you believe exists between each of the above pairs of frequencies. Also compare the consonance between each of these two pairs of frequencies with those pairs shown on the previous page. Finally, rank the four pairs of frequencies in the order you believe them to be in from most to least consonant and explain your ranking.

RANKING OF FREQUENCY RATIOS FROM MOST TO LEAST CONSONANT

Frequency Ratio	Consonance
	Most
	Least

EXPLANATION OF RANKING

A SURVEY OF HISTORIC MUSICAL SCALES

YOU SHOULD NOW be able to verify two things about musical scales: they are closely tied to the physics of waves and sound and they are not trivial to design. Many scales have been developed over time and in many cultures. The simplest have only four notes and the most complex have dozens of notes. Three of the most important scales are examined here: the Pythagorean Scale, the Scale of Just Intonation, and the Even Temperament Scale.

THE PYTHAGOREAN SCALE

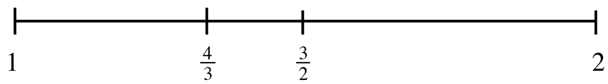
It has already been mentioned that Pythagoras discovered the consonance between two strings (or frequencies, as we know now) whose lengths were in the ratio of two small integers. The best consonance was heard with the 2:1 and 3:2 ratios. So Pythagoras started with two strings, one twice as long as the other and the tones from these strings were defined as the highest and lowest tones in his scale. To produce intermediate tones, he used ratios of string lengths that were 3:2. To get this ratio a string length could be multiplied by 3/2 or divide by 3/2. In either case, the original string and the new string would have length ratios of 3:2. There's one catch though. In multiplying or dividing a length by 3/2, the new length might be shorter than the shortest string or longer than the longest string. However, this really isn't a problem since a length that is either too long or too short can simply be cut in half or doubled in length (even repeatedly) in order to get it into the necessary range of lengths. This may seem a bit cavalier, but remember that strings that differ by a ratio of 2:1 sound virtually the same. This sounds more complicated than it really is. It's easier to understand by simply watching the manner in which Pythagoras created his scale.

Let's give the shortest string in Pythagoras' scale a length of 1. That means the longer one has a length of 2. These two strings, when plucked, will produce frequencies that sound good together. To get the first two additional strings we'll multiply the shorter length by 3/2 and divide the longer length by 3/2:

$$1 \bullet \frac{3}{2} = \frac{3}{2}$$

$$2 / \frac{3}{2} = 2 \bullet \frac{2}{3} = \frac{4}{3}$$

In order of increasing length, the four string lengths are 1, 4/3, 3/2, 2. These also represent the numbers you would multiply the first frequency in your musical scale by in order to get the three additional frequencies. These are represented (to scale) in the following graph.



If 100 Hz were chosen as the first note in this simple musical scale, the scale would consist of the following frequencies: 100 Hz, 133 Hz, 150 Hz, and 200 Hz. This simple four-note scale is thought to have been used to tune the ancient lyre. Pythagoras however expanded the notes of this simplistic scale by creating two new string lengths from the intermediate lengths, 4/3 and 3/2, making sure that the string length ratio continued to be 3:2.

$$\frac{4}{3} / \frac{3}{2} = \frac{4}{3} \bullet \frac{2}{3} = \frac{8}{9}$$

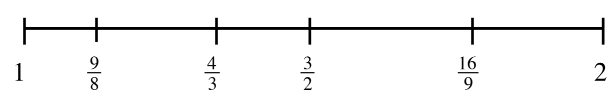
$$\frac{3}{2} \bullet \frac{3}{2} = \frac{9}{4}$$

Neither 8/9 nor 9/4 are between 1 and 2 so they must be adjusted by either multiplying or dividing these lengths by two (recall this type of adjustment will lead to tones that sound virtually the same).

$$\frac{8}{9} \bullet 2 = \frac{16}{9}$$

$$\frac{9}{4} / 2 = \frac{9}{8}$$

The string lengths (or frequency multipliers) in order of increasing size is now 1, 9/8, 4/3, 3/2, 16/9, 2. These are represented (to scale) in the following graph.

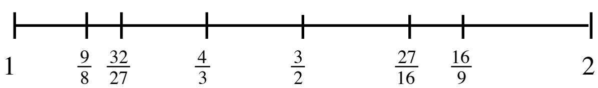


This scale, with five different notes, is known as the **pentatonic scale** and has been very popular in many cultures, especially in Eastern music. But modern Western music is based on seven different notes, so we'll go through the process once more, adjusting the last two lengths so that there are two additional lengths in a 3:2 ratio.

$$\frac{16}{9}/\frac{3}{2}=\frac{16}{9}\bullet\frac{2}{3}=\frac{32}{27}$$

$$\frac{9}{8}\bullet\frac{3}{2}=\frac{27}{16}$$

The final collection of string lengths (or frequency multipliers) in order of increasing size is now 1, 9/8, 32/27, 4/3, 3/2, 27/16, 16/9, 2. These are represented (to scale) in the following graph.



This is one version of the **Pythagorean Scale** (other ways of building the scale lead to a different order of intervals). Whether you think of the intervals on the graph as string lengths or as frequencies, you can easily verify with a pocket calculator that there are many ratios of small integers meaning that combinations of many of the frequencies lead to high levels of consonance. You should also notice that the intervals are not uniform. There are mostly larger intervals, but also two smaller intervals (between the second and third note and between the sixth and seventh note). In every case, to get from one frequency to the next across a large interval, the first frequency must be multiplied by 9/8 (for example, $32/27\cdot 9/8=4/3$ *and* $4/3\cdot 9/8=3/2$). In both cases of moving across the smaller intervals, the first frequency must be multiplied by 256/243. Lets look at these two intervals more carefully:

$$\frac{9}{8}=1.125 \quad and \quad \frac{256}{243}=1.053$$

Looking at the decimal representations of these two intervals shows that the larger is an increase of a little over 12% and the smaller is an increase of just less than half that (5.3%). The larger of these intervals is known as a **whole tone**, *W* and the smaller is known as a **semitone**, *s* (*semi*, because it's about half the increase of a whole tone). (*Whole tones* and *semitones* are synonymous with the terms *whole steps* and *half steps*). Going up Pythagoras' scale then requires a series of different increases in frequency:

W s W W W s W

If you were to start with a certain frequency, calculate all the others in the scale and then play them in succession it would sound like the familiar "*Do – Re – Me – Fa – So – La – Ti – Do"* except that it would start with *Ray*. The more familiar string of tones beginning with *"Doe"* (the note "C") would then have to have the following increases in frequency intervals:

	W	*W*	*s*	*W*	*W*	*W*	*s*
C_1	**D**	**E**	**F**	**G**	**A**	**B**	**C_2**
Do	*Re*	*Me*	*Fa*	*So*	*La*	*Ti*	*Do*

This is familiar to musicians, but most non-musicians have probably not noticed the smaller increase in pitch when going from "*Me*" to "*Fa*" and from "*Ti*" to "*Do*." Try it. Do you hear the difference in the intervals?

There are seven different notes in the Pythagorean scale (eight if you include the last note, which is one diapson higher than the first note, and thus essentially the same sound as the first). In this scale, the eighth note has a ratio of 2:1 with the first note, the fifth note has a ratio of 3:2 with the first note, and the fourth note has ratio of 4:3 with the first note. This is the origin of the musical terms the **octave** the **perfect fifth**, and the **perfect fourth**. (Click on the interval name to hear two notes separated by that interval.) This is why, for example, "G" sounds good when played with either the upper or the lower "C." It is a fifth above the lower C and a fourth below the upper C.

C_i	**D**	**E**	**F**	**G**	**A**	**B**	**C_f**
1	$\frac{9}{8}$	$\frac{81}{64}$	$\frac{4}{3}$	$\frac{3}{2}$	$\frac{27}{16}$	$\frac{243}{128}$	2

Table 4.1: The Pythagorean Scale Intervals for a C major scale. Multiplying the frequency of a particular "C" by one of the fractions in the table gives the frequency of the note above that fraction.

It's also interesting to look at the ratios between individual notes. Table 4.2 shows the full list of frequency intervals between adjacent tones.

Note change	Frequency ratio
C → D	9/8
D → E	9/8
E → F	256/243
F → G	9/8
G → A	9/8
A → B	9/8
B → C	256/243

Table 4.2: Pythagorean Scale interval ratios. Note there are two possible intervals between notes: 9/8 (a whole tone) and 256/243 (a semitone).

Do you get it? (4.1)

Assuming C_5 is defined as 523 Hz, determine the other frequencies of the Pythagorean scale. Show work below and fill in the table with the appropriate frequencies.

Note	**C_5**	**D**	**E**	**F**	**G**	**A**	**B**	**C_6**
Freq. (Hz)	523							

THE JUST INTONATION SCALE

The scale of **Just Intonation** or **Just Scale** also has a Greek origin, but this time not from a mathematician, but from the spectacular astronomer, Ptolemy. Like Pythagoras, Ptolemy heard consonance in the frequency ratios of 2:1, 3:2, and 4:3. He also heard consonance in the frequency ratio of 5:4. He found that groups of three frequencies sounded particularly good together when their ratios to each other were 4:5:6. His method of generating the frequency intervals of what we now know as the *C major* scale was to group the notes of that scale into triads each having the frequency ratios of 4:5:6.

4	5	6
C_i	E	G
G	B	D
F	A	C_f

Let's start with C_i and give it a value of 1. That automatically makes C_f = 2. In order to get the C_i:E:G frequency ratios to be 4:5:6 we can represent C_1 as 4/4. Then E would be 5/4 and G would be 6/4, or 3/2. So now we have the first three frequency ratios:

$\mathbf{C_i}$	**D**	**E**	**F**	**G**	**A**	**B**	$\mathbf{C_f}$
1		$\frac{5}{4}$		$\frac{3}{2}$			2

With the next triad (G,B,D) we'll start with G = 3/2 and multiply this ratio by 4/4, 5/4, and 6/4 to get the next set of 4:5:6 frequency ratios.

$$G = \frac{3}{2} \bullet \frac{4}{4} = \frac{12}{8} = \frac{3}{2}$$

$$B = \frac{3}{2} \bullet \frac{5}{4} = \frac{15}{8}$$

$$D = \frac{3}{2} \bullet \frac{6}{4} = \frac{18}{8} = \frac{9}{4}$$

This last value for D is more than double the frequency for C_1, so we have to divide it by two to get it back within the octave bound by C_i and C_f. Therefore, $D = (9/4)/2 = 9/8$. Now we have two more frequency ratios:

C_i	D	E	F	G	A	B	C_f
1	$\frac{9}{8}$	$\frac{5}{4}$		$\frac{3}{2}$		$\frac{15}{8}$	2

With the last triad (F,A,C_f) we'll have to start with C_f because it's the only one we know the value of. In order to the get the next set of 4:5:6 frequency ratios, this time we'll have to multiply C_f's value by 4/6, 5/6, and 6/6.

$$F = 2 \bullet \frac{4}{6} = \frac{8}{6} = \frac{4}{3}$$

$$A = 2 \bullet \frac{5}{6} = \frac{10}{6} = \frac{5}{3}$$

$$C_f = 2 \bullet \frac{6}{6} = \frac{12}{6} = 2$$

The complete scale of frequency ratios is:

C_i	D	E	F	G	A	B	C_f
1	$\frac{9}{8}$	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{3}{2}$	$\frac{5}{3}$	$\frac{15}{8}$	2

Table 4.3: Just Scale Intervals for a C major scale. Multiplying the frequency of a particular "C" by one of the fractions in the table gives the frequency of the note above that fraction.

It's also interesting to look at the ratios between individual notes. For example, to get from C to D, the C's frequency must be multiplied by 9/8, but to get from D to E, D's frequency must be multiplied by 10/9. Table 4.4 shows the full list of frequency intervals between adjacent tones.

Note change	Frequency ratio
C □ D	9/8
D □ E	10/9
E □ F	16/15
F □ G	9/8
G □ A	10/9
A □ B	9/8
B □ C	16/15

Table 4.4: Just Scale interval ratios. Note there are three possible intervals between notes: 9/8 (a major whole tone), 10/9 (a minor whole tone), and 16/15 (a semitone).

The largest of these ratios, 9/8 (12.5% increase), is the same as the Pythagorean whole tone. In the Just scale it is known as a ***major* whole tone**. There is another fairly large interval, 10/9 (11.1% increase), known as a ***minor* whole tone**. The smallest, interval, 16/15 (6.7% increase), while slightly different from the smallest Pythagorean interval, is also called a **semitone**.

Some of the names of small integer ratios of frequencies that produce consonance have already been discussed. These and others are listed, in order of decreasing consonance, in the Table 4.5.

Frequency Ratio	Interval	Interval name
2/1	C → C	Octave
3/2	C → G	Perfect fifth
4/3	C → F	Perfect fourth
5/3	C → A	Major sixth
5/4	C → E	Major third
8/5	E → C	Minor sixth
6/5	A → C	Minor third

Table 4.5: Just Scale interval ratios and common names. Click on any of the names to hear the interval played.

Tables 4.2 and 4.4 give the interval ratios between notes within one octave of either the Pythagorean or the Just scales. However, if the frequencies within a particular octave are too low or too high, they can all be adjusted up or down one or more octaves, either by starting with the first "C" of the desired octave and multiplying by each of the frequency ratios or simply by multiplying each frequency in the above scale by the appropriate integer (this integer would be 2 if you wanted to produce the next higher octave).

Do you get it? (4.2)

C_4 is the frequency or note one octave below C_5 (523 Hz). Calculate the frequencies of the notes in the Just scale within this octave.

Note	C_4	D_4	E_4	F_4	G_4	A_4	B_4	C_5
Freq. (Hz)								

A PROBLEM WITH TRANSPOSING MUSIC IN THE PYTHAGOREAN AND JUST SCALES

One of the problems with both the Pythagorean and the Just scales is that songs are not easily transposable. For example, if a song were written in the key of *C* (meaning that it starts with the note, *C*) and you wanted to change it so that it was written in the key of *F*, it wouldn't sound right. It wouldn't be as easy as transposing all the notes in the song up by three notes (*C* → *F*, *F* → *B*, etc.) because of the difference in intervals between various notes. Let's say the first two notes of the song you want to transpose are *C* and *F* and you want to rewrite the song in the key of *F*. Using Table 4.2, the increase in frequency for the first note (*C* → *F*) is:

$$\frac{9}{8}\bullet\frac{9}{8}\bullet\frac{256}{243}=1.33.$$

If the newly transposed song is to have the same basic sound, then every note in the song should change by that same interval, but going from *F* → *B* is actually an interval of:

$$\frac{9}{8}\bullet\frac{9}{8}\bullet\frac{9}{8}=1.42.$$

The Just scale has the same problem. One attempt at a solution would be to add extra notes (sharps and flats) between the whole tones on both scales so that there were always choices for notes between the whole tones if needed for transposing. So going from *C* to *F* or from *F* to *B* would be an increase of five semitones in both cases. But there is still the problem of the two different whole tone intervals in the Just scale. And in both scales, there is the problem that the semitone intervals are not exactly half the interval of the whole tones, so the idea of creating additional notes between the whole tones doesn't fully solve the problem of being able to seamlessly transpose music. There are also mistuned interval issues as well with both scales that will be dealt with later, but first we'll look at the perfect solution to the transposing problem – the Equal Tempered scale.

THE EQUAL TEMPERAMENT SCALE

The Equal Temperament scale attempts to correct the frequency spacing problem without losing the benefits of the special intervals within the two previous scales. It includes the seven notes of the previous scales and adds five sharps (for a total of 12 semitones), but it places them so that **the ratio of the frequencies of any two adjacent notes is the same**. This is not as simplistic as it may seem at first glance. It is not the same as taking the frequency interval between two C's and dividing it by 12. This would not lead to the condition that two adjacent frequencies have the same ratio. For example:

$$\frac{13/12}{12/12} = \frac{13}{12} \quad but \quad \frac{14/12}{13/12} = \frac{14}{13} \quad and \quad \frac{13}{12} \neq \frac{14}{13}$$

Instead, it means that multiplying the frequency of a note in the scale by a certain number gives the frequency of the next note. And multiplying the frequency of this second note by the same number gives the frequency of the note following the second, and so on. Ultimately, after going through this process twelve times, the frequency of the twelfth note must be an octave higher – it must be twice the frequency of the first note. So if the multiplier is "r", then:

$$r \cdot r \cdot r \cdot r \cdot r \cdot r \cdot r \cdot r \cdot r \cdot r \cdot r \cdot r = 2$$

$$\Rightarrow \quad r^{12} = 2 \quad \Rightarrow \quad r = \sqrt[12]{2} = 1.05946$$

Click here to see a table of all frequencies of the notes in the Equal Temperament Scale.

Example

The note, D, is two semitones higher than C. If C_6 is 1046.5 Hz, what is D_7 on the Equal Temperament Scale?

Solution:

- Think about the problem logically in terms of semitones and octaves.

  D_7 is one octave above D_6, so if D_6 can be found then its frequency just needs to be doubled. Since D_6 is two semitones higher than C_6, its frequency must be multiplied twice by the Equal temperament multiplier.

- Do the calculations.
  1. $D_6 = 1046.5Hz \times (1.05946)^2$
     $= 1174.7Hz$

     $\Rightarrow D_7 = 2D_6 = 2(1174.7)Hz$

     $= \boxed{2349.3Hz}$

     or alternately

     $D_7 = 1046.5Hz \times (1.05946)^{14}$

     $= \boxed{2349.3Hz}$

With the extensive calculations in this section it would be easy to lose sight of what the purpose of the last few pages was. Let's step back a bit. You know music when you hear it. It is a fundamentally different sound than what we call noise. That's partly because of the rhythm associated with music, but also because the tones used sound good when played together. Physically, multiple tones sound good together when the ratio of their frequencies is a ratio of small numbers. There are many scales that endeavor to do this, including the Just scale, the Pythagorean scale, and Equal Temperament scale, all discussed above. In Western music the Equal Temperament scale is the most widely used. Its twelve semitones all differ in a ratio of $\sqrt[12]{2}$ from each adjacent semitone. All notes of the major scale are separated by two semitones except for E and F, and B and C. These two pairs are separated by one semitone. The $^{\#}$ indicates a sharp (see Table 4.6).

1	2	3	4	5	6	7	8	9	10	11	12
C	C#	**D**	D#	**E**	**F**	F#	**G**	G#	**A**	A#	**B**

Table 4.6: The notes of the 12-tone Equal Temperament scale. All notes are separated by two semitones (a whole tone) except for *E* and *F*, and *B* and *C*. The # indicates a "sharp."

Now remember the ancient discovery of frequency ratios of small numbers leading to pleasant sounds when the two frequencies are played together. This has not been entirely sacrificed for the purpose of mathematical expediency. It turns out that $\left(\sqrt[12]{2}\right)^7 = 1.498$ which is about 0.1% different from 1.5 or 3/2 (the perfect fifth). So in the Equal Temperament scale, the seventh semitone note above any note in the scale will be close to a perfect fifth above it. That means C still sounds good with the G above it, but D# also sounds just as good when played with the A# above it (see figure 4.3). The next best sounding combination, the perfect fourth, occurs when two notes played together have a frequency ratio of 4/3. It conveniently turns out that $\left(\sqrt[12]{2}\right)^5 = 1.335$ (less than 0.4% different from a perfect fourth). So the fifth semitone higher than any note will be higher by a virtual perfect fourth (see figure 4.3).

Do you get it? (4.3)

In one suggestion for a standard frequency, C_4 is 256 Hz. In this particular standard, what would be the frequency of E_4 on each of the following scales?

a. Pythagorean:

b. Just:

c. Equal Temperament

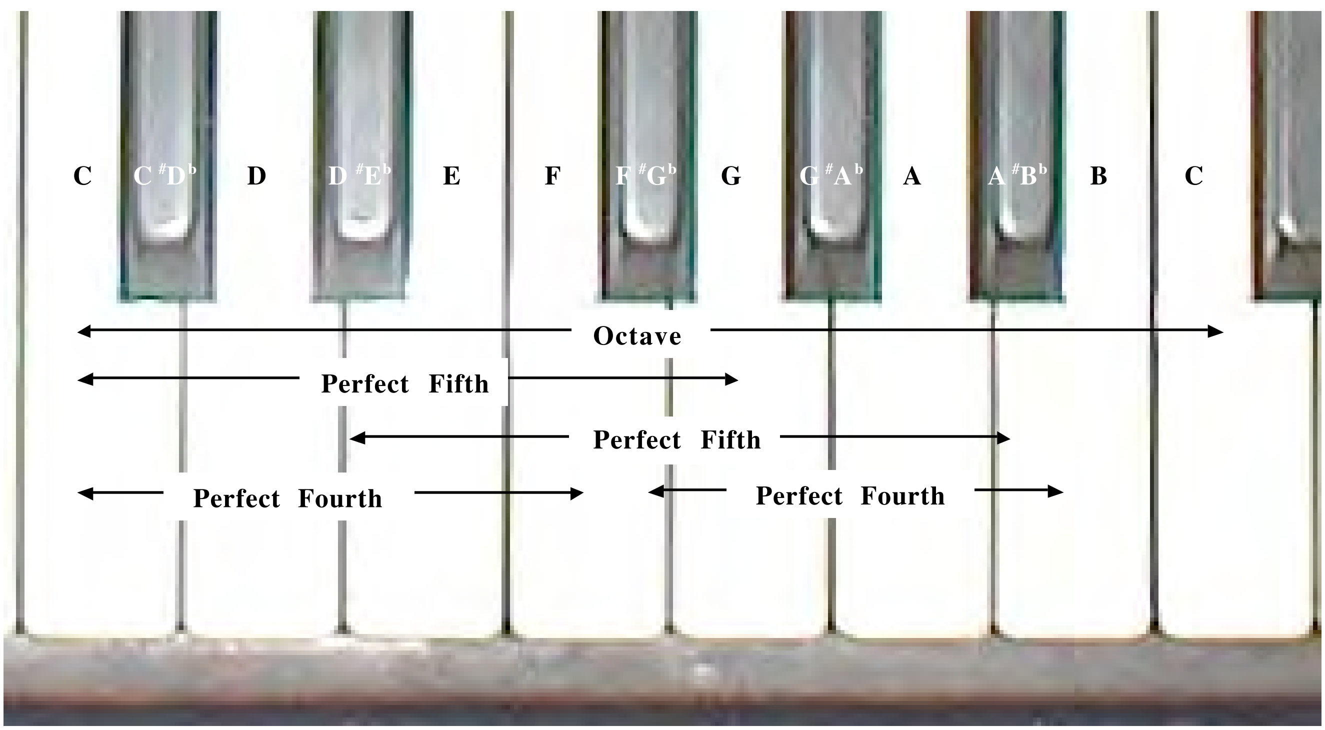


Figure 4.3: The chromatic scale with various consonant musical intervals

A CRITICAL COMPARISON OF SCALES

An important question is how large can the percentage difference from perfect intervals be before the difference is detected or unacceptable? A more precise method for comparing frequencies is with the unit, **cents**. Musically, one cent is 1/100 of an Equal Temperament semitone. And since there are 12 semitones per octave, one cent is 1/1200 of an octave. But remember that this is still a ratio of frequencies, so it's not as simple as saying that one cent is equivalent to 2 divided by 1200. Rather, any two frequencies that differ by one cent will have the same frequency ratio. So after the frequency ratio "*R*" of one cent is multiplied by itself 1,200 times, the result is 2.

$$\left(R_{(1cent)}\right)^{1200} = 2$$

$$\Rightarrow R_{(1cent)} = 2^{\frac{1}{1200}} = 1.000578$$

The frequency ratio equivalent to two cents would be:

$$\Rightarrow R_{(2cents)} = \left(2^{\frac{1}{1200}}\right)\left(2^{\frac{1}{1200}}\right) \Rightarrow R_{(2cents)} = 2^{\frac{2}{1200}}$$

Finally, the frequency ratio of *I* cents would be:

$$R = 2^{\frac{I}{1200}}$$

And the corresponding number of cents for a particular frequency ratio can be found by taking the logarithm of both sides of this equation:

$$\log R = \log\left(2^{\frac{I}{1200}}\right) \Rightarrow \log R = \frac{I}{1200}\log 2$$

$$\Rightarrow I = \frac{1200\log R}{\log 2}$$

Example

The perfect fifth is a frequency ratio of 1.5. How many cents is this and how does it compare with the fifths of the Just, Pythagorean, and Equal Tempered scale?

Solution:

- Do the calculations.

$$I = \frac{1200\log R}{\log 2} = \frac{1200\log(1.5)}{\log 2} = \boxed{702 cents}$$

- Make the comparisons.

  Just scale: identical.

  Pythagorean scale: identical.

  Equal Tempered scale: 2 cents less.

Of course, all of this is only useful if there is some understanding of how many cents a musical scale interval can differ from one of the perfect intervals before it becomes detected or unacceptable. With that in mind, a musician with a good ear can easily detect a mistuning of 5 cents and a 10 to 15 cent deviation from perfect intervals is enough to be unacceptable, although at times musicians will often intentionally deviate by this much for the purpose of artistic interpretation. Table 4.7 compares several important ideal musical intervals with their equal tempered approximations.

Interval	Frequency ratio	Frequency ratio (cents)	Equal Temp. scale (cents)
Octave	2 : 1	1200	1200
Fifth	3 : 2	702	700
Fourth	4 : 3	498	500
Major sixth	5 : 3	884	900
Major third	5 : 4	386	400
Minor sixth	8 : 5	814	800
Minor third	6 : 5	316	300

Table 4.7: A comparison of ideal intervals with their approximations on the Equal Tempered scale. Note the large deviations from ideal for the thirds and sixths.

Because of its equal intervals, the Equal Temperament scale makes transposing music very simple. For example, someone with only a marginal understanding of the scale could easily take a melody written in the key of C and rewrite it in the key of F by simply increasing every note's frequency by five semitones. This is a major advantage over the scales with unequal intervals. But a close look at Table 4.7 illustrates why many musicians take issue with the approximations necessary to make these intervals equal. Although its octave is perfect and its fifth and fourth differ from the ideal by only 2 cents, the equal tempered sixths and thirds all differ from the ideal anywhere from 14 to 16 cents, clearly mistuned and noticeable by anyone with a good ear.

The frequency interval unit of cents is especially useful in analyzing the musical scales. In the following example, Table 4.2 is redrawn with the frequency intervals expressed in cents rather than fractions. This leads to a quick way to analyze the quality of the various consonant intervals. Using $I = 1200 \log R / \log 2$ to convert the intervals gives:

Notes	Frequency interval (cents)
C_i	0
D	204
E	408
F	498
G	702
A	906
B	1110
C_f	1200

Table 4.8: Pythagorean scale interval ratios expressed in cents

It's clear that the interval between C_i and G is 702 cents – a perfect fifth. It's almost as clear to see that the interval between D and A $(906 cents - 204 cents = 702 cents)$ is also a perfect fifth. The additive nature of the cents unit makes it easy to judge the quality of various intervals. Table 4.8 shows the values of various important intervals in the Pythagorean scale.

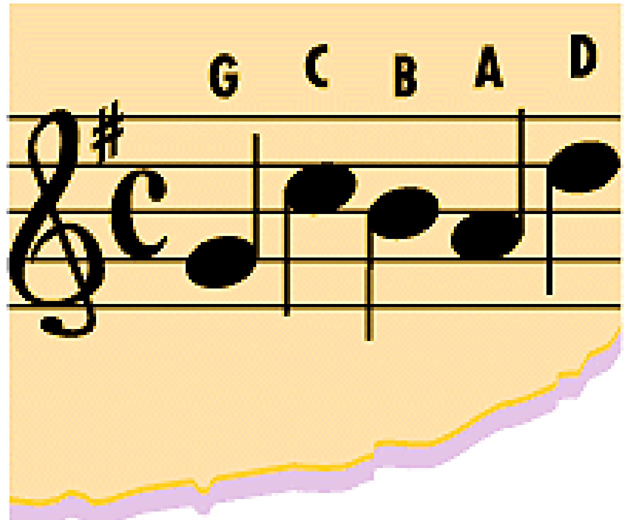
Interval	Interval name	Frequency ratio (cents)
C_i – C_f	Octave	1200
C_i – G	Fifth	702
D – A	Fifth	702
E – B	Fifth	702
F – C	Fifth	702
C_i – F	Fourth	498
D – G	Fourth	498
E – A	Fourth	498
G – C_f	Fourth	498
C_i – E	Major third	408
F – A	Major third	408
G – B	Major third	408

Table 4.9: An evaluation of the Pythagorean intervals. Note the abundance of perfect fifths and fourths, but also the very poorly tuned thirds.

Table 4.9 shows why the Pythagorean scale is so important. Within the major scale, there are four perfect fifths and four perfect fourths (there are many more of both if the entire chromatic scale is used). However, the three major thirds differ by 22 cents $(408 cents - 386 cents)$ from the perfect major third, noticeably sharp to most ears. This is the reason that Pythagoras felt the major third was dissonant. The minor third is a problem in the Pythagorean scale as well. Going from E to G is an increase of 294 cents (see Table 4.8), but the perfect minor third, 6/5, is 316 cents (see Table 4.7). So the Pythagorean major third is 22 cents sharp and the minor third is flat by the same amount. This 22-cent interval is actually 21.5 cents (due to rounding errors) and is known as the **syntonic comma**, □. One way to deal with this particular mistuning is to compromise the position of the E. Decreasing it a bit would help the consonance of both the major third (C to E) and the minor third (E to G). This and similar adjustments made to other notes in the scale are known as **meantone tuning**. There are different types of meantone tuning, but the most popular appears to be **quarter-comma meantone tuning**. In this version, every note, except for C, is adjusted by either 1/4, 2/4, 3/4, 4/4, or 5/4 of the syntonic comma (see Table 4.10)

Pythagorean Note	Quarter-comma Meantone adjustment
C	none
D	[illegible] $\frac{2}{4}$ [illegible]
E	[illegible]
F	$+\frac{1}{4}$ [illegible]
G	[illegible] $\frac{1}{4}$ [illegible]
A	[illegible] $\frac{3}{4}$ [illegible]
B	[illegible] $\frac{5}{4}$ [illegible]
C	none

Table 4.10: Quarter-comma Meantone adjustments to the Pythagorean scale.



"If I were not a physicist, I would probably be a musician. I often think in music. I live my daydreams in music. I see my life in terms of music."
– Albert Einstein

"When I hear music, I fear no danger. I am invulnerable. I see no foe. I am related to the earliest times, and to the latest."
– Henry David Thoreau

ACTIVITY

CREATE A MUSICAL SCALE

Now that you've had some time to look at various musical scale constructions as well as to consider the merits of these various scales, it's time to try your hand at it. In this activity, you will design an equal-tempered musical scale. Your goal is to discover one that has both a reasonable number of total intervals as well as many consonant intervals. You may choose any number of intervals except for the commonly used twelve intervals.

1. What is the number of intervals in your equal-tempered scale? _________

2. What is the size of your equal-tempered interval? Express this as a decimal interval increase as well as the number of cents per interval.

3. In the space below, indicate the presence of the following consonant intervals in your scale: fifth, fourth, major third, minor third, major sixth, and minor sixth. State how many of your equal-tempered intervals are equivalent to these consonant intervals. (Do not consider that a particular consonant interval exists unless it is within 0.01 of the perfect interval.)

Fifth:

Fourth:

Major third:

Minor third:

Major sixth:

Minor sixth:

4. Evaluate each of the consonant intervals in your scale, considering a 5-cent or less deviation to be perfect and a 6 – 15 cent deviation to be acceptable.

5. Compare the quality of your equal tempered scale to that of the 12 tone equal tempered scale. Consider both the presence and quality of the consonant intervals as well as the number of total intervals.

ACTIVITY

EVALUATING IMPORTANT MUSICAL SCALES

INTRODUCTION

You've seen that the process of building or choosing a particular musical scale is not trivial. It should be clear now that there is no such thing as perfect tuning. This was shown for the 12-tone Equal Temperament scale in Table 4.7 and for the Pythagorean scale in Table 4.9. During this activity you will be asked to evaluate the pros and cons of three types of scales: the Just Scale, the Quarter-comma Meantone scale, and variants of the Equal Tempered Scale.

CALCULATIONS (EXPLAIN THE PROCESS YOU'RE USING THROUGHOUT AND SHOW ALL CALCULATIONS CLEARLY)

THE JUST SCALE

1. Transform Table 4.3 so that the Just scale frequency intervals are expressed in cents.

Just scale notes	Frequency interval (cents)
C_i	
D	
E	
F	
G	
A	
B	
C_f	

2. Now create a table in which you identify all fifths, fourths, major thirds and minor thirds in the Just scale. What are the best and worst aspects of the Just scale when compared to the Pythagorean scale and 12-tone Equal Tempered scale?

THE QUARTER-COMMA MEANTONE SCALE

3. Use Tables 4.8 and 4.10 to create a table showing the frequency intervals in the Quarter-comma Meantone scale

Quarter-comma Meantone scale notes	Frequency interval (cents)
C_i	
D	
E	
F	
G	
A	
B	
C_f	

4. Now create a table in which you identify all fifths, fourths, major thirds and minor thirds in the Quarter-comma Meantone scale. What are the best and worst aspects of the Quarter-comma Meantone scale when compared to the Pythagorean scale, Just scale, and 12-tone Equal Tempered scale?

OTHER EQUAL TEMPERED SCALES

The 12-tone Equal Temperament scale is a compromise between the consonance of perfect intervals and the utility of equal intervals. Some have wondered why the fifth and the fourth work so well, but the other intervals don't. The reason is simply that in deciding to use 12 tones, the spacing between some notes just happens to be very close to that of the consonant intervals. Other equal interval temperaments have been proposed over time, including those with 19, 31, and 53 tones. Remember that although these have 19, 31, and 53 notes, respectively, the range from the first note until the note following the last note is still one octave (1200 cents). The first step is to determine the interval between notes in the three scales.

5. What is the interval (in cents) between notes on each of the following equal temperament scales?
   a. **19 tone**
   b. **31-tone**
   c. **53-tone**

6. Now determine the number of steps in each of these Equal Temperament scales required for the following intervals: perfect fifth, perfect fourth, major third, major sixth, minor third, and minor sixth. Finally, indicate the deviation of each of these intervals from the ideal interval as shown in column 3 of Table 4.7.

19-tone Equal Temperament

Interval	Number of steps on the scale	Deviation from ideal interval (cents)
Perfect Fifth		
Perfect Fourth		
Major third		
Major sixth		
Minor third		
Minor sixth		

31-tone Equal Temperament

Interval	Number of steps on the scale	Deviation from ideal interval (cents)
Perfect Fifth		
Perfect Fourth		
Major third		
Major sixth		
Minor third		
Minor sixth		

53-tone Equal Temperament

Interval	Number of steps on the scale	Deviation from ideal interval (cents)
Perfect Fifth		
Perfect Fourth		
Major third		
Major sixth		
Minor third		
Minor sixth		

7. Rank the three Equal Tempered scales against each other in terms of the quality of their tuning of the major consonant intervals.

8. Thinking of the piano keyboard, explain carefully what the chief problem of these Equal Temperament scales are.

9. What are the best and worst aspects of the best of the three Equal Tempered scales when compared to the Quarter-comma Meantone scale, Pythagorean scale, Just scale?

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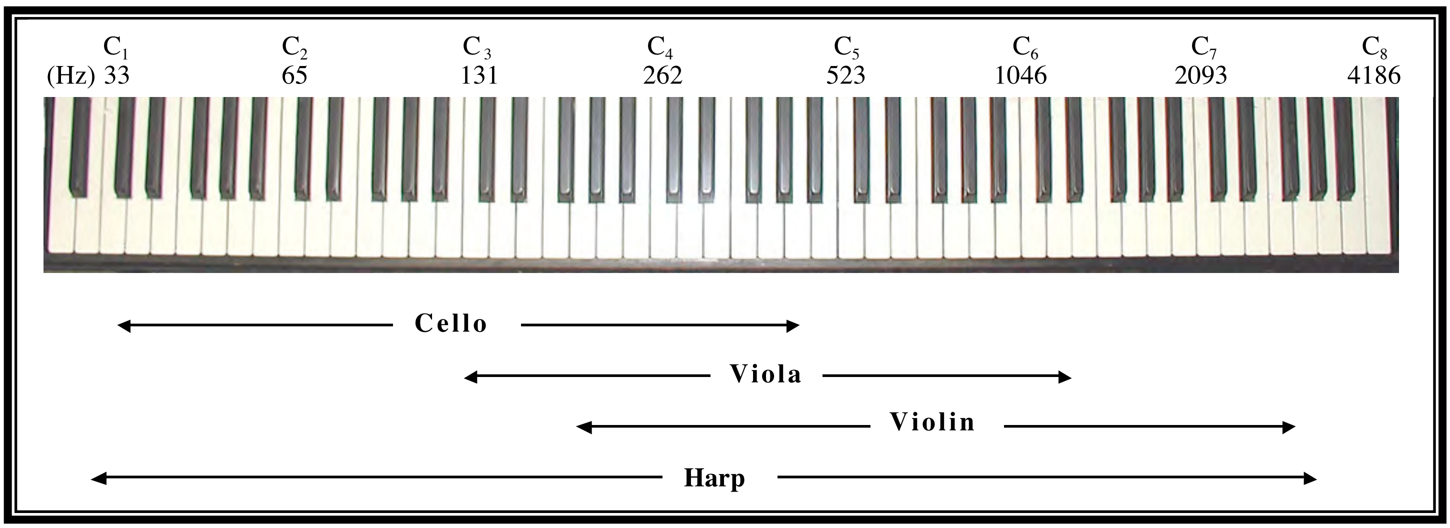
"Words make you think a thought. Music makes you feel a feeling. A song makes you feel a thought."
– E.Y. Harbug

CHAPTER 5
CHORDOPHONES
(STRINGED INSTRUMENTS)



THE YEAR WAS 1968, the event was Woodstock, one of the bands was The Grateful Dead, the man was Jerry Garcia, and the musical instrument he used to make rock and roll history was the guitar – a chordophone (stringed instrument). Instruments in this class are easy to pick out. They have strings, which either get plucked (like guitars), bowed (like violins), or thumped (like pianos). **It includes all instruments whose standing wave constraint is that at each end of the medium there must be a node**. Technically this includes drums, but because of the two dimensional nature of the vibrating medium, the physics becomes a lot more complicated. We'll just look at true strings.

THE FIRST MODE

The simplest way a string can vibrate in a standing wave condition is with the two required nodes at the ends of the string and an antinode in the middle of the string (see Figure 5.1).

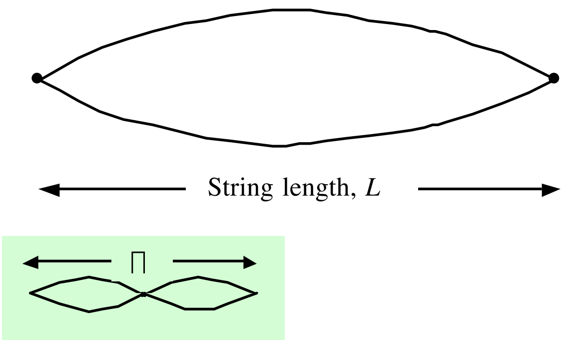


Figure 5.1: First mode of vibration. This is the simplest way for a string to vibrate in a standing wave condition. This mode generates the fundamental frequency.

This is the first mode. The length of the string (in wavelengths) is half a wavelength (see Figure 5.1). That means that for the string length, L:

$$L = \frac{1}{2}\lambda \quad \Rightarrow \quad \lambda = 2L.$$

Now frequency, in general, can be found using $f = v/\lambda$, so for the first mode of a stringed instrument:

$$f_1 = \frac{v}{2L}.$$

WAVE SPEEDS ON STRINGS

Wave speed on strings depends on two factors: the tension in the string and the "linear mass density" of the string. Tightening or loosening the strings with the tuners can change their **tension**. It takes more force to pluck a taut string from its resting position. And with more force acting on the taut string, the more quickly it will restore itself to its unplucked position. So, more tension means a quicker response and therefore, a higher velocity for the wave on the string.

The wave velocity can also be affected by the "heaviness" of the string. Strictly speaking, it's the **linear mass density** of the string that causes this effect. Linear mass density is the amount of mass per length of string (in kg/m). The greater this density, the greater the overall mass of a particular string will be. Most people have noticed that the strings on a guitar vary in thickness. The thicker strings have greater mass, which gives them more inertia, or resistance to changes in motion. This greater inertia causes the thicker, more massive strings to have a slower response after being plucked – causing a lower wave velocity.

It should be clear that higher tension, T, leads to higher wave speed (see Figure 5.2), while higher linear mass density, μ, leads to lower wave speed (see Figure 5.3). The two factors have opposite effects on the string's wave velocity, v. This is clear in the equation for string wave velocity:

$$v = \sqrt{\frac{T}{\mu}}$$

Recall the frequency for the first mode on a string is $f_1 = v/2L$. Combining this with the expression for the string's wave velocity, the fundamental frequency of a stringed instrument becomes:

$$f_1 = \frac{\sqrt{\frac{T}{\mu}}}{2L}$$

This complicated looking equation points out three physical relationships that affect the fundamental frequency of a vibrating string. Since string tension is in the numerator of the equation, frequency has a direct relationship with it – if tension increases, so does frequency. However, since both linear mass density and length are in denominators of the equation, increasing either of them decreases the frequency. The following graphic illustrates these three relationships

$$\begin{array}{lcl} T\uparrow & \Rightarrow & f\uparrow \\ \mu\uparrow & \Rightarrow & f\downarrow \\ L\uparrow & \Rightarrow & f\downarrow \end{array}$$

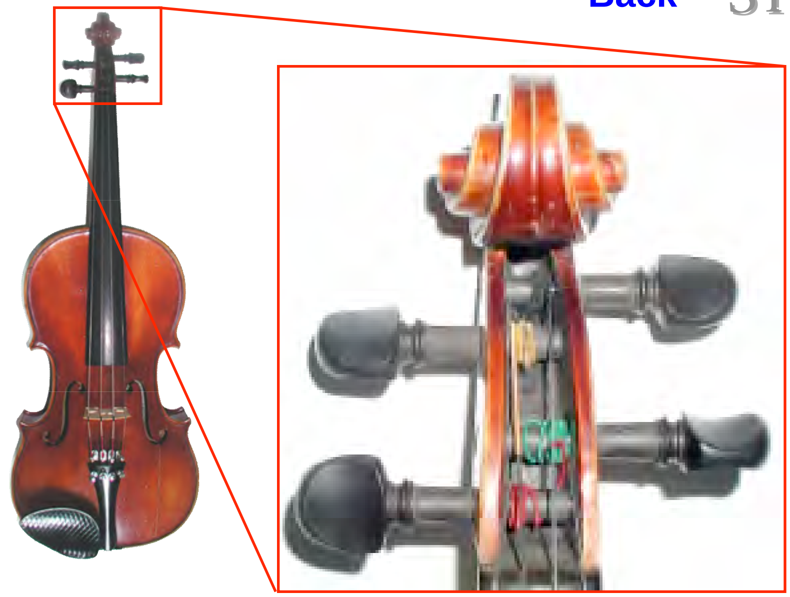


Figure 5.2: Changing a string's tension changes its frequency of vibration. When a tuner either tightens or loosens a string on a violin its frequency of vibration changes. The equation for the fundamental frequency of a vibrating string, $f_1 = \left(\sqrt{T/\mu}\right)/2L$, shows the connection between string tension and frequency. Since tension is in the numerator of the square root, if it increases, so will the string's frequency.

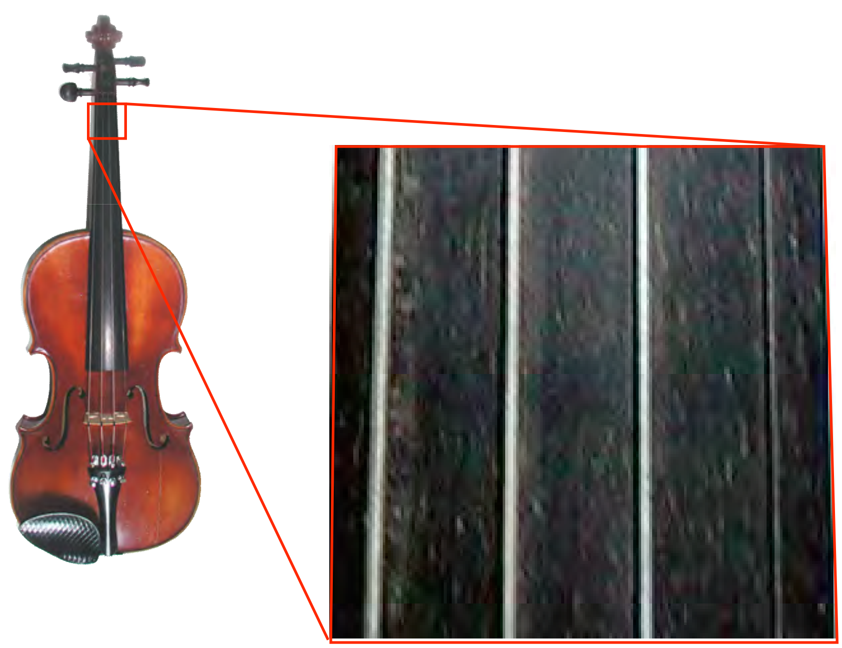


Figure 5.3: Changing a string's linear mass density, μ, changes its frequency of vibration. Two strings with the same tension, but different mass will vibrate with different frequency. The equation for the fundamental frequency of a vibrating string, $f_1 = \left(\sqrt{T/\mu}\right)/2L$, shows the connection between linear mass density and frequency. Since linear mass density is in the denominator of the square root, if it increases, the string's frequency decreases. The thicker and heavier strings on the violin are the ones that play the lower frequency notes.

THE SECOND MODE

Now lets look at the next possible mode of vibration. It would be the next simplest way that the string could vibrate in a standing wave pattern with the two required nodes at the end of the string (see Figure 5.4)

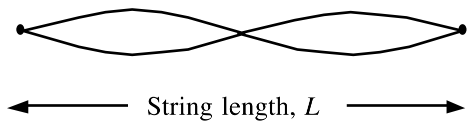


Figure 5.4: Second mode of vibration. This is the next simplest way for a string to vibrate in a standing wave condition. This mode generates the first overtone.

We can figure out the frequency of the second mode the same way as before. The only difference is that the string length is now equal to the wavelength of the wave on the string. So, for the frequency of the second mode of a string:

$$f = \frac{v}{\lambda} \Rightarrow f_2 = \frac{\sqrt{\frac{T}{\mu}}}{L}$$

You should notice that this is exactly twice the frequency of the first mode, $f_2 = 2f_1$. And, when you pluck the guitar string, both modes are actually present (along with many even higher modes, as discussed earlier).

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Do you get it? (5.1)

a. In the space below, draw the string vibrating in the third mode:

b. Now write the equation for the frequency of the third mode. Explain how you arrived at this equation.

c. In the space below, draw the string vibrating in the fourth mode:

d. Now write the equation for the frequency of the fourth mode. Explain how you arrived at this equation.

e. Finally, look for a pattern in these four frequencies and write the equation for the n^{th} mode frequency.

Figure 5.5: A two-stringed musical instrument from Java with very large tuners. See moviebelow.

SOUND PRODUCTION IN STRINGED INSTRUMENTS

Figure 5.6: The energy used to pluck the strings of the guitar is transferred to the top of the guitar as the strings vibrate against the bridge.

If you were to remove a string from any stringed instrument (guitar, violin, piano) and hold it taut outside the window of a moving car, you would find very little resistance from the air, even if the car were moving very fast. That's because the string has a very thin profile and pushes against much less air than if you held a marimba bar of the same length outside the car window. If you stretched the string in between two large concrete blocks and plucked it you would hear very little sound. Not only would the string vibrate against very little air, but also because of the huge impedance difference between the string and the concrete blocks, its vibrations would transmit very little wave energy to the blocks. With so little of its energy transmitted, the string would simply vibrate for a long time, producing very little sound. For the non-electric stringed instrument to efficiently produce music, its strings must couple to some object (with similar impedance) that will vibrate at the same frequency and move a lot of air. To accomplish this, the strings of guitars, violins, pianos, and other stringed acoustic instruments all attach in some fashion to a *soundboard*. Figure 5.6 shows the strings of a guitar stretched between the *nut* and the *bridge*. Figure 5.7 shows a magnified image of the bridge attachment to the top of the guitar. As a string vibrates it applies a force to the top of the guitar, which varies with the frequency of the string. Since the impedance change between the string and the bridge is not so drastic as that between the string and the concrete blocks, much of the wave energy of the string is transmitted to the bridge and guitar top, causing the vibration of a much greater surface area. This, in turn, moves a tremendously greater amount of air than the string alone, making the vibration clearly audible.

Electric stringed instruments use a different method for amplification. Try strumming an electric guitar with its power turned off and the sound produced will be similar to that of the taut string between the two concrete blocks. But turn on the power and the sound is dramatically increased. The string of an electric guitar vibrates over the top of up to three electromagnetic *pickups*. The pickup consists of a coil of wire with a magnetic core. As the string (which must be made of steel) vibrates through the pickup's magnetic field, it changes the flux of the magnetic field passing through the core. And since the flux change is at the same frequency as the vibrating string, this becomes a signal, which can be amplified through a loud speaker. The string is not coupled directly to any soundboard, and will thus vibrate for a far longer time than that of its acoustic cousin.

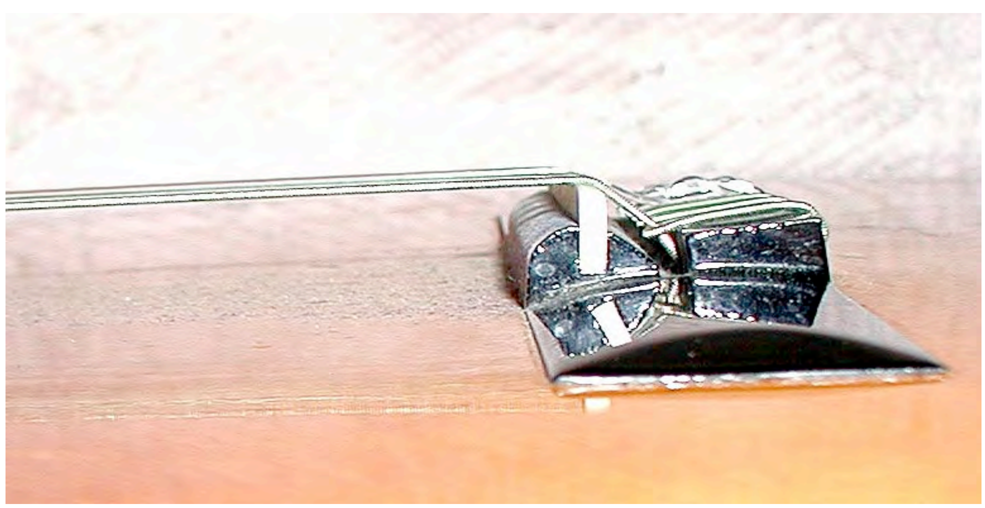
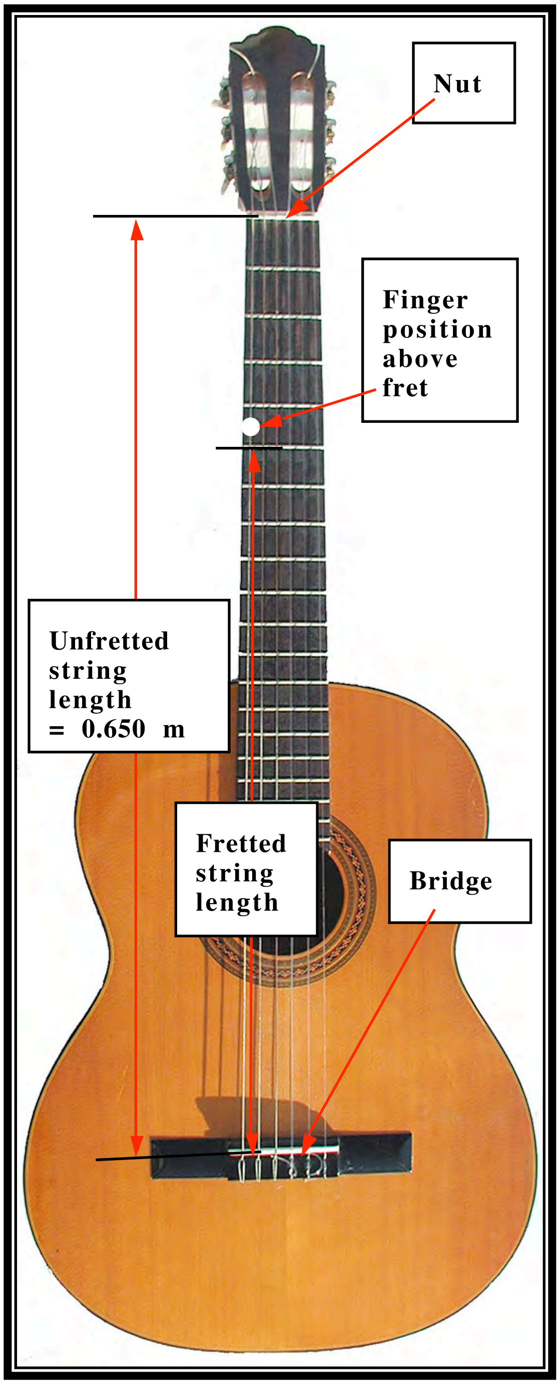


Figure 5.7: Tension from the stretched guitar strings causes a downward force on the bridge, which moves the entire face of the guitar at the same frequency as that of the strings. This larger movement of air greatly amplifies the nearly inaudible sound of the strings alone.

INVESTIGATION

THE GUITAR



INTRODUCTION

It is believed that the origin of the guitar was in Egypt more than 3,000 years ago. The modern guitar pictured above has its strings tuned to E_2, A_2, D_3, G_3, B_3, and E_4. In order to get intermediate frequencies, the strings are "fretted." Pressing a finger down on the space above the fret changes the length of a particular string with negligible change to the tension in the string. The new string length is measured from the fret to the bridge. The length of the unfretted strings (from nut to bridge) is 0.65 m.

CALCULATIONS (EXPLAIN THE PROCESS YOU'RE USING THROUGHOUT AND SHOW ALL CALCULATIONS CLEARLY)

1. The frequency of C_2 is 65.406 Hz. Use this frequency and the even temperament scale to calculate the frequencies of all the unfretted strings

   E_2:

   G_3:

   A_2:

   B_3:

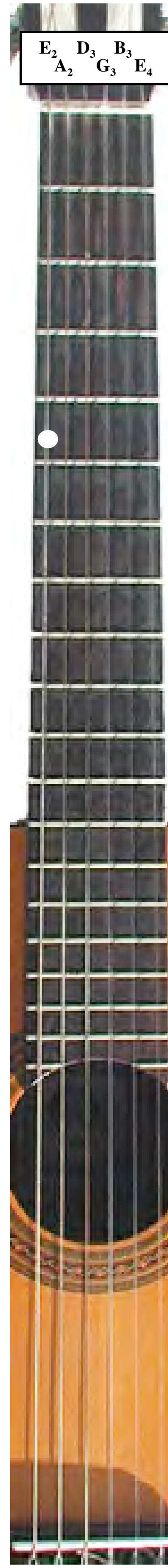
   D_3:

   E_4:

2. Give some justification, based on the idea of consonant intervals, for choosing these particular notes for the tuning of the six strings on the guitar.

3. Calculate the speed of the second harmonic wave on the E_2 string.

4. The linear mass density of the A_2 string is $0.0085 kg/m$. Calculate the tension in this string.



5. Make measurements on the photograph to the right to calculate the frequency of the E_2 string fretted as shown to the right. You should notice that the frequency is now the same as A_2 string. How close are you to A_2?

6. If you had a tuning fork that had the frequency of E_2, you could use it to make sure your E_2 string was tuned perfectly. If the E_2 string were out of tune by say, 3 *Hz*, you would hear a beat frequency of 3 *Hz* when sounded together with the tuning fork. Then you could tighten or loosen the string until no beats were heard. What could you do then to make sure the A_2 string was tuned correctly?

7. Now calculate the length for the E_2 string and the A_2 string to have the frequency of D_3. On the photograph to the right, show where you would fret both strings (indicate in the same way as shown on the E_2 string).

8. Finally, calculate the string lengths and corresponding fret positions for the:
   a. D_3 string to have the frequency of G_3.

   b. G_3 string to have the frequency of B_3.

   c. B_3 string to have the frequency of E_4.

9. a. How many semitones higher is A_2 from E_2? __________

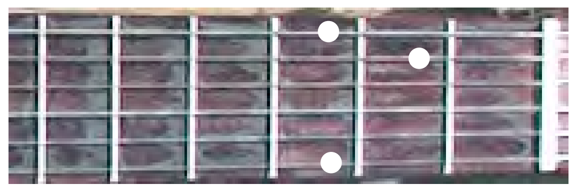
b. How many frets did you have to go down on the E_2 string to get the frequency of A_2? __________

c. Use your answers to "a" and "b" to make a statement about how you think the placement of frets is determined. As evidence for your hypothesis, include information from your answers in problems 7 and 8.

The photographs in the following three questions show popular chords guitar players use. The white dots still represent the places where strings are fretted. All strings are strummed except those with an "**x**" over them.

10. a. List the notes played when this chord is strummed:

_____ _____ _____ _____ _____ _____



E_2
A_2
D_3
G_3
B_3
E_4

b. Why does this chord sound good when its notes are played?

11. a. List the notes played when this chord is strummed:

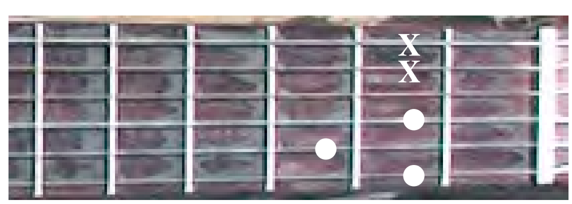
_____ _____ _____ _____ _____ _____

E_2
A_2
D_3
G_3
B_3
E_4

b. Why does this chord sound good when its notes are played?

12. a. List the notes played when this chord is strummed:

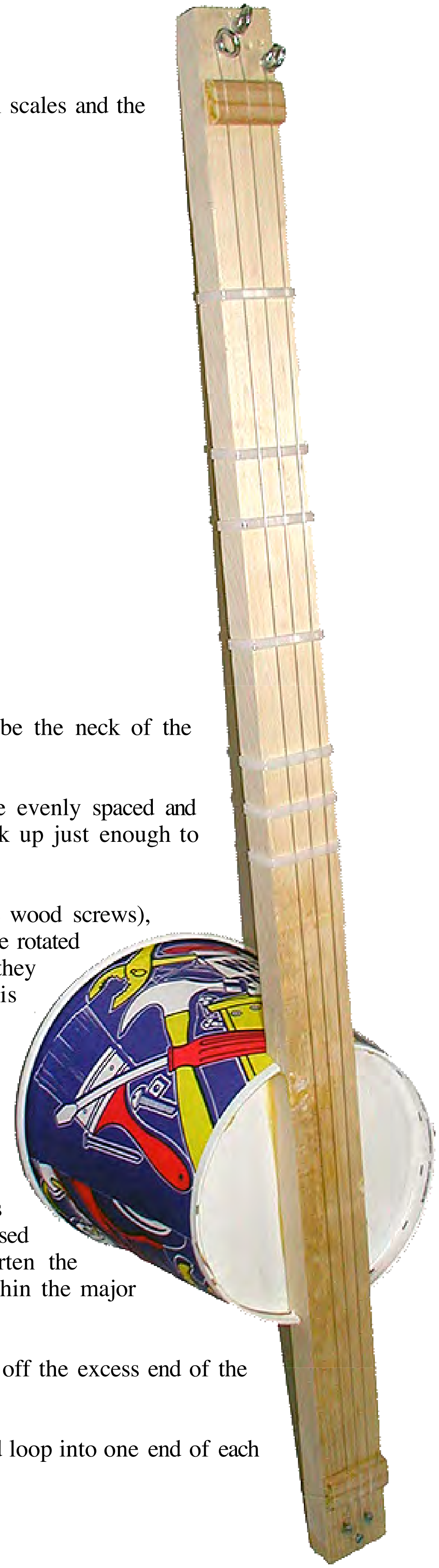
_____ _____ _____ _____



E_2
A_2
D_3
G_3
B_3
E_4

b. Why does this chord sound good when its notes are played?

BUILDING A THREE STRINGED GUITAR



OBJECTIVE:

To design and build a three stringed guitar based on the physics of musical scales and the physics of the vibrating strings.

MATERIALS:

- One 80 cm piece of 1” x 2” pine
- Approximately 10 cm of thin wood molding
- Three #4 x 3/4” wood screws
- Three #14 screw eyes
- Approximately 3 meters of 30 - 40 lb test fishing line
- One disposable 2 - 4 quart paint bucket
- Seven thin 8” plastic cable ties
- Small wood saw
- Screwdriver
- Small pair of pliers
- Wood glue
- Metric ruler or tape

PROCEDURE

1. Sand the corners and edges of the 80 cm piece of pine. This will be the neck of the guitar.

2. Screw the wood screws into one end of the neck. They should be evenly spaced and approximately 2 cm from the end of the board. The heads should stick up just enough to get a loop of fishing line around them.

3. Put the screw eyes into the other end of the board (in line with the wood screws), making sure that they do not interfere with each other when they are rotated (they will need to be staggered). Screw them in only enough so that they are stable. They will be tightened later when the fishing line is attached.

4. Cut two 4-cm long pieces of molding. Glue each of these to the neck 3-cm in from each of the two sets of screws (on the side of the screws closer to the middle of the neck). These are the nut and the bridge.

5. Measure the distance in between the nut and the bridge. Use this distance along with the intervals of one of the musical scales discussed to calculate the placement of frets. The frets will be used to shorten the lengths of the strings so that they can produce all the frequencies within the major scale of one octave.

6. Place a cable tie at each fret position. Tighten it with pliers and cut off the excess end of the cable tie.

7. Cut strings from the fishing line in 1-meter increments. Tie a knotted loop into one end of each string.

8. For each string, place the knotted loop over the wood screw and pull the other end tight, wrapping it clockwise several times around the appropriate screw eye. While maintaining the tension in the string, loop it through the screw eye and tie a knot.

9. If necessary, cut notches in the bottom of the paint bucket edges so that the neck can lie flush against the paint bucket bottom. Position the paint bucket about equidistant between the bridge and the fret closest to the bridge. Glue the bottom of the paint bucket to the neck. Let dry.

10. Choose the string that will produce the lowest frequency. Rotate the eyehook screw attached to it (clockwise) while plucking the string in order to achieve the desired tone.

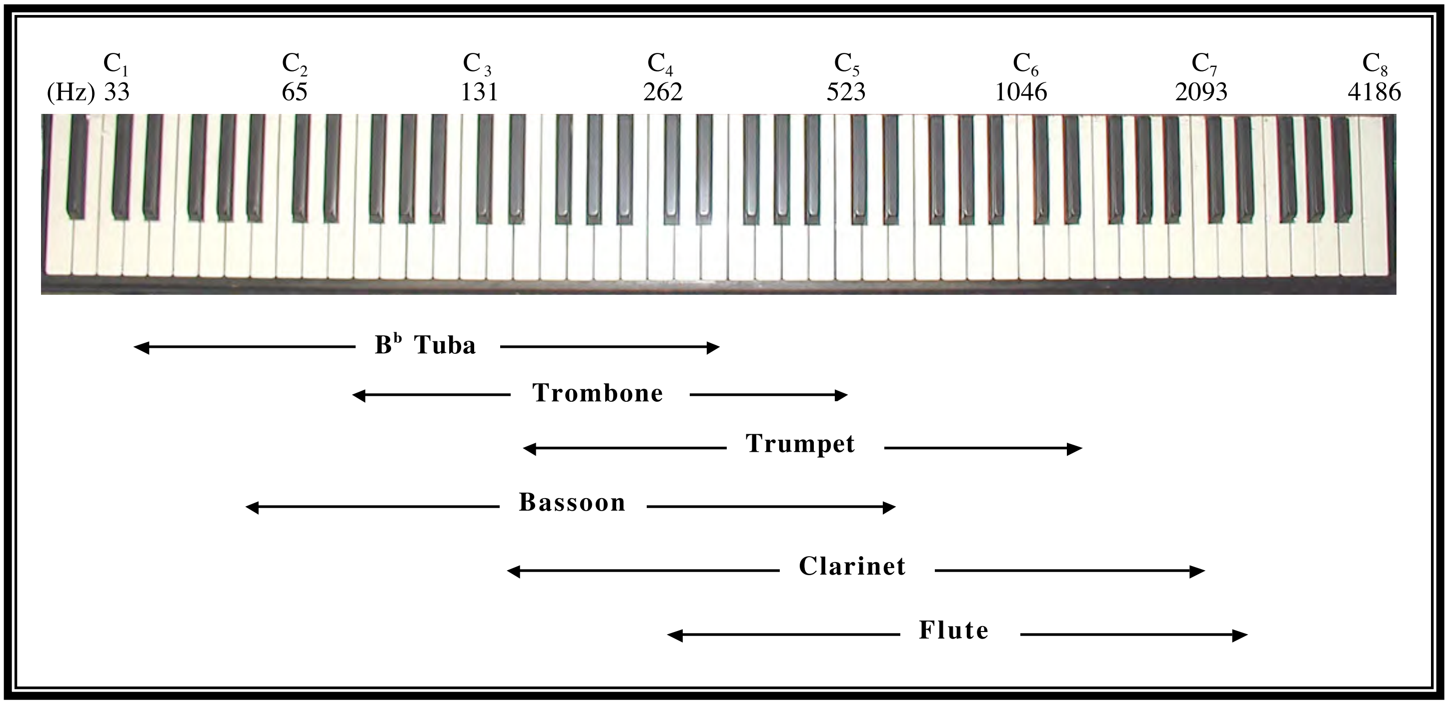
11. Fret the lowest frequency string so that it is a fifth higher than its fundamental frequency. Using beats, tune the next string to this frequency.

12. Fret the second string so that it is a fourth higher than its fundamental frequency. Again use beats to tune the third string to this frequency.

13. Play the guitar. With the open strings you can play an octave, a fifth, and a fourth. You can also play tunes and other intervals by using the frets.

"The Irish gave the bagpipes to the Scotts as a joke, but the Scotts haven't seen the joke yet."
– Oliver Herford

CHAPTER 6
AEROPHONES
(WIND INSTRUMENTS)



STANDING WAVES ARE created in the column of air within wind instruments, or aerophones. Most people have a more difficult time visualizing the process of a wave of air reflecting in a flute or clarinet as opposed to the reflection of a wave on the string of a guitar. On the guitar, for example, it's easy to picture a wave on one of its strings slamming into the bridge. The bridge represents a wave medium with obviously different impedance than that of the string, causing a significant reflection of the wave. It may not be as obvious, but when the wave of air in a wind instrument reaches the end of the instrument, and all that lies beyond it is an open room, it encounters an impedance change every bit as real as the change seen by the wave on the guitar string when it reaches the bridge. The openness beyond the end of the wind instrument is a less constricted environment for the wave (lower impedance), and because of this change in impedance, a portion of the wave must be reflected back into the instrument.

To initially create the sound wave within the aerophone, the player directs a stream of air into the instrument. This air stream is interrupted and chopped into airbursts at a frequency within the audible range. The interruption is accomplished by vibrating one of three types of reeds: a mechanical reed, a "lip reed," or an "air reed."

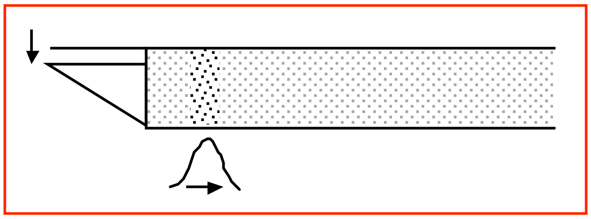
THE MECHANICAL REED

Instruments like the clarinet, oboe, saxophone, and bagpipes all have mechanical reeds that can be set into vibration by the player as he forces an air stream into the instrument. Most of us have held a taut blade of grass between the knuckles of our thumbs and then blown air through the gaps on either side of the grass blade. If the tension on the grass is just right and the air is blown with the necessary force, the grass will start shrieking. The air rushing by causes a standing wave to be formed on the grass blade, the frequency of which is in the audible range. You can change the pitch of the sound by moving the thumbs a bit so that the tension is varied.

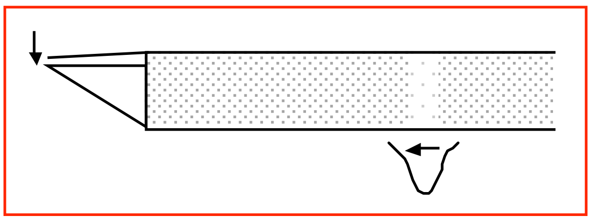
The mechanical reeds in wind instruments (see Figure 6.2) can be set into vibration like the grass blade, except that the length of the tube largely governs the frequency of the reed. Figure 6.3 illustrates the vibration mechanism for the mechanical reed/tube system.

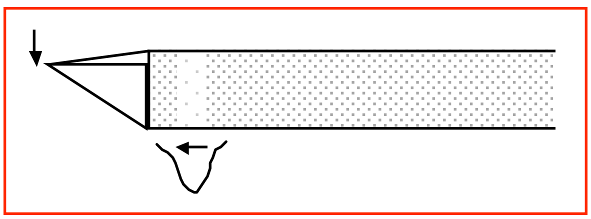
Figure 6.1: The clarinet is in the woodwind class of aerophones. The player blows across a small reed, which causes the reed to vibrate. The vibrating reed allows bursts of air into the body of the clarinet. The bursts of air are responsible for the resulting standing wave of air that becomes the distinctive sound produced by the clarinet.

Figure 6.2: A clarinet mouthpiece and reed. By holding her mouth in just the right position and with just the right tension, the clarinet player causes the reed to vibrate up and down against the mouthpiece. Each time the reed rises, creating an opening above the mouthpiece, a burst of air from the player enters the clarinet. The length of the clarinet largely controls the frequency of the reed's vibration.

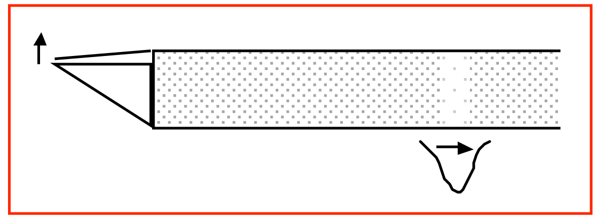


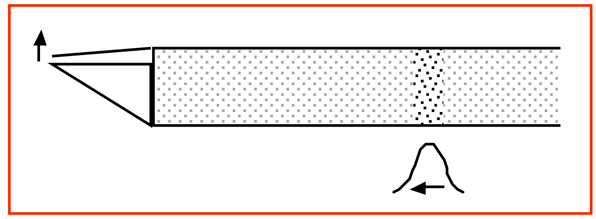
1. The puff of air the player initially blows through the instrument begins to pull the reed toward the body of the instrument and creates a region of high pressure that moves toward the end of the instrument.



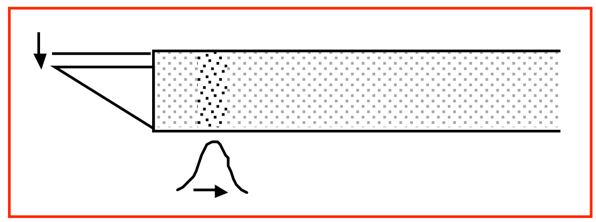


2. When the high-pressure region reaches the lower, normal air pressure at the end of the instrument it is largely reflected. This causes the reflected pulse to be a negative or low-pressure pulse and has the effect of pulling the reed toward the body of the instrument, closing the gap sharply.





3. The low-pressure pulse is reflected from this closed end of the instrument and moves back to the other end. When it reaches the higher, normal air pressure, it largely reflects again, this time as a high-pressure pulse.



4. When the high-pressure pulse reaches the reed, it forces it open and allows the air from the player to enter and reverse the direction of the high-pressure pulse. This pattern of feedback makes it easy for the player to keep the reed frequency at the same frequency as that of the pressure wave inside the instrument.

FIGURE 6.3: VIBRATION MECHANISM FOR THE MECHANICAL REED/TUBE SYSTEM

LIP AND AIR REEDS

Not all wind instruments use a mechanical reed. Brass instruments like the trumpet, trombone, and tuba use a "lip reed" (see Figure 6.4). Although the lips are not true reeds, when the player "buzzes" his lips on the mouthpiece of the instrument they cause the air stream to become interrupted in the same way as the mechanical reed does. The same type of feedback occurs as well, with low-pressure portions of the sound wave pulling the lips closed and high-pressure portions forcing the lips open so that another interrupted portion of the air stream can enter the instrument.

Figure 6.4: While the trumpet player's lips are not a true "reed," when they buzz against the mouthpiece, they provide the same frequency of airbursts as a mechanical reed.

The last method for interrupting the air stream of a wind instrument is with an "air reed" (see Figure 6.5). As the player blows a steady air stream into the mouthpiece of the recorder, the air runs into the sharp edge just past the hole in the top of the mouthpiece. The air stream gets split and a portion of the air enters the recorder, moving down the tube and reflecting back from its open end, as in the case of other wind instruments. However, rather than interrupting the air stream mechanically with a wooden reed or with the lips of the player, the reflected air pulse itself acts as a reed. The low and high-pressure portions of this sound wave in the recorder interrupt the player's air stream, causing it to oscillate in and out of the instrument at the same frequency as the standing sound wave. Other wind instruments that rely on the "air reed" include flutes, organ pipes and even toy whistles. This, by the way, is the mechanism that people use when they use their lips to whistle a tune.

Figure 6.5: The "air reed." A portion of the air stream entering the recorder moves down the tube and reflects back from its open end, as in the case of other wind instruments. However, rather than interrupting the air stream mechanically with a mechanical reed or with the lips of the player, the reflected air pulse itself acts as a reed. The low and high-pressure portions of this sound wave in the recorder interrupt the player's air stream, causing it to oscillate in and out of the instrument at the same frequency as the standing sound wave.

Regardless of the type of reed used, wind instruments all create sound by sustaining a standing wave of air within the column of the instrument. The other major distinction between wind instruments is whether there are two ends open (open pipes) or only one end open (closed pipes).

OPEN PIPE WIND INSTRUMENTS

Recorders and flutes are both examples of open pipe instruments because at both ends of the instrument there is an opening through which air can move freely. Since the air at both ends of the column is relatively free to move, the standing wave constraint for this class is that **both ends of the air column must be a displacement antinode**.

The simplest way a column for air in an open pipe to vibrate in a standing wave pattern is with the two required antinodes at the ends of the pipe and a node in the middle of the pipe (see Figure 6.6). This is the first mode of vibration.

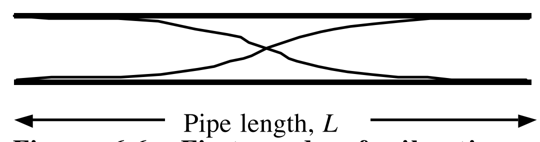


Figure 6.6: First mode of vibration. This is the simplest way for a column of air to vibrate (in an open pipe) in a standing wave condition. This mode generates the fundamental frequency.

The length of the pipe (in wavelengths) is $1/2\lambda$ (think of it as two quarters joined at the ends). Therefore:

$$L = \frac{1}{2}\lambda \Rightarrow \lambda = 2L \text{ (same as for strings).}$$

We can find the frequency like we did before by using $f = v/\lambda$. Thus, for the first mode of an open pipe instrument:

$$f_1 = \frac{v}{2L}.$$

The speed, v, of waves in the pipe is just the speed of sound in air, much simpler than that for the string. The frequency of a particular mode of an open pipe depends only on the length of the pipe and the temperature of the air.

Now let's look at the next possible mode of vibration. It is the next simplest way that the column of air can vibrate in a standing wave pattern with the two required antinodes at the ends of the pipe (see Figure 6.7).

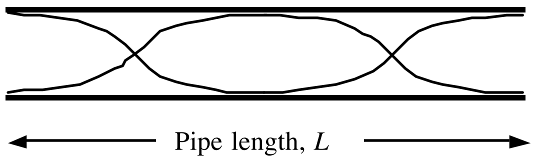


Figure 6.7: Second mode of vibration. This is the next simplest way for a string to vibrate in a standing wave condition. This mode generates the first overtone.

We can figure out the frequency of this second mode the same way as before. The only difference is that the pipe length is now equal to one wavelength of the sound wave in the pipe. So for the frequency of the second mode of an open pipe:

$$f = \frac{v}{L} \Rightarrow f_2 = \frac{v_{sound}}{L}.$$

You should notice that, as with the modes of the string, this is exactly twice the frequency of the first mode, $f_2 = 2f_1$.

Do you get it? (6.1)

a. In the space below, draw the air vibrating in the third mode:

b. Now write the equation for the frequency of the third mode. Explain how you arrived at this equation.

c. In the space below, draw the air vibrating in the fourth mode:

d. Now write the equation for the frequency of the fourth mode. Explain how you arrived at this equation.

e. Now look for a pattern in these four frequencies and write the equation for the n^{th} mode frequency. Explain how you arrived at this equation.

CLOSED PIPE WIND INSTRUMENTS

The trumpet and the clarinet are both examples of closed pipe wind instruments, because at one end the player's lips prevent the free flow of air. Since the air at the open end of the column is relatively free to move, but is constrained at the closed end, the standing wave constraint for closed pipes is that **the open end of the air column must be a displacement antinode and the closed end must be a node.**

Figure 6.8: The Panpipe is a closed pipe instrument popular among Peruvian musicians.

The simplest way a column of air in a closed pipe can vibrate in a standing wave pattern is with the required antinode at the open end and the required node at the closed end of the pipe (see Figure 6.9). This is the fundamental frequency, or first mode.

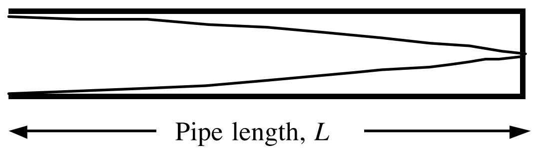


Figure 6.9: First mode of vibration. This is the simplest way for a column of air to vibrate (in a closed pipe) in a standing wave condition. This mode generates the fundamental frequency.

The length of the pipe (in wavelengths) is $(1/4)\lambda$. So, for the first mode of a closed pipe:

$$L = \frac{1}{4}\lambda \quad \Rightarrow \quad \lambda = 4L.$$

We can find the frequency as we did before by using $f = v/\lambda$. Thus, for the first mode of a closed pipe instrument:

$$f_1 = \frac{v_{sound}}{4L}.$$

Now let's look at the next possible mode of vibration. It is the next simplest way that the column of air can vibrate in a standing wave pattern with the required antinode at the open end and the required node at the closed end of the pipe (see Figure 6.10).

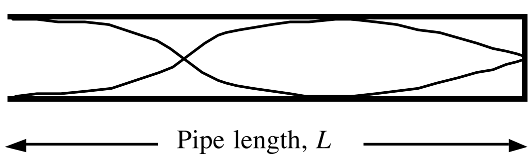


Figure 6.10: Second mode of vibration. This is the next simplest way for a string to vibrate in a standing wave condition. This mode generates the first overtone.

We can figure out the frequency of this next mode the same way as before. The pipe length is now equal to $3/4$ the wavelength of the sound wave in the pipe:

$$L = \frac{3}{4}\lambda \quad \Rightarrow \quad \lambda = \frac{4}{3}L.$$

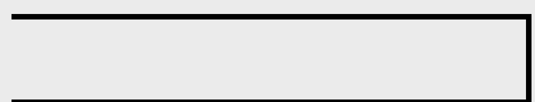
And the frequency of the next mode of a closed pipe is:

$$f = \frac{v}{\lambda} \quad \Rightarrow \quad f = \frac{v_{sound}}{\frac{4}{3}L} \quad \Rightarrow \quad f = \frac{3v_{sound}}{4L}.$$

You should notice a difference here between the modes of strings and open pipes compared to the modes of closed pipes. This second mode is three times the frequency of the fundamental, or first mode. This means that this harmonic is the *third* harmonic. The second harmonic can't be produced with the standing wave constraints on the closed pipe. This is actually true for all the even harmonics of closed *cylindrical* pipes. However, if the closed pipe has a *conical* bore or an appropriate flare at the end (like the trumpet), the spectrum of harmonics continues to be similar to that of an open pipe.

Do you get it? (6.2)

a. In the space below, draw the air vibrating in the mode after the third:



b. Now write the equation for the frequency of the next higher mode after the third. Explain how you arrived at this equation.

c. In the space below, draw the air vibrating in the mode two higher than the third:

d. Now write the equation for the frequency of the mode two higher than the third. Explain how you arrived at this equation.

e. Now look for a pattern in these four frequencies and write the equation for the n^{th} mode frequency. Explain how you arrived at this equation.

THE END EFFECT

Everything said about open and closed pipes is basically true so far. However, there is one little issue that needs to be dealt with. Otherwise, the music you make with any aerophone you personally construct will be *flat* – the frequency will be too low. A musician with a good ear could tell there was a problem. The problem is with the open ends of these pipes. When the standing wave in the column of air reaches a closed end in a pipe there is a hard reflection. However, when the same standing wave reaches the open end of a pipe, the reflection doesn't occur so abruptly. It actually moves out into the air a bit before reflecting back. This makes the pipes acoustically longer than their physical length. This "end effect" is equal to 61% of the radius of the pipe. This end effect must be added to the length of the closed pipe and added twice to the length of the open pipe.

Example

Let's say you wanted to make a flute from one-inch PVC pipe. If the lowest desired note is C_5 on the Equal Temperament Scale (523.25 Hz), what length should it be cut?

Solution:

- *Identify all givens (explicit and implicit) and label with the proper symbol.*

Given: f_1 = 523.25 Hz
n = 1 (Lowest frequency)
v = 343 m/s (no temperature given)

$$r = (0.5inch)\left(\frac{2.54cm}{1inch}\right)$$
$$= 1.27cm = .0127m$$

- *Determine what you're trying to find.*

Length is specifically asked for
Find: L

- *Do the calculations.*

$$1.\ f_1 = \frac{v}{2L_{acoust.}} \Rightarrow L_{acoust.} = \frac{v}{2f_1} = \frac{343m/s}{2\left(523.5\frac{1}{s}\right)}$$

$$= 0.328m$$

2. 0.328 m is the desired acoustic length of the pipe, which includes the end effect on both ends of the pipe. Therefore, the pipe must be cut shorter than 0.328 m by two end effects.

$$L_{phys.} = L_{acoust.} - 2(end\ \ effect)$$

$$\Rightarrow L_{phys.} = 0.328m - 2(.61 \times .0127m)$$

$$= \boxed{0.312m}$$

Do you get it? (6.3)

You want to make a 4.0-cm diameter tube, closed at one end, that has a fundamental frequency of 512 Hz and the temperature is 30° C.

a. What length will you cut the tube?

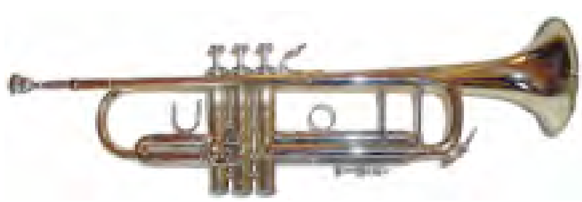
b. If you blew across the tube a lot harder to produce the next mode, what would be the new frequency?

c. Now if the bottom of the tube were cut off so that the tube was open at both ends, what would be the new fundamental frequency?

CHANGING THE PITCH OF WIND INSTRUMENTS

In equations for both open and closed pipe wind instruments, the variables that can change the frequency are the number of the mode, the speed of sound in air, and the length of the pipe. It would be difficult or impossible to try to control the pitch of an instrument by varying the temperature of the air, so that leaves only the number of the mode and the length of the pipe as methods for changing the pitch. Some wind instruments, like the bugle, have a single, fixed-length tube. The only way the bugle player can change the pitch of the instrument is to change the manner in which he buzzes his lips, and so change the mode of the standing wave within the bugle. The standard military bugle is thus unable to play all the notes in the diatonic scale. It typically is used to play tunes like *taps* and *reveille*, which only require the bugle's third through sixth modes: G_4, C_5, E_5, G_5. In order to play all notes in the diatonic or chromatic scale, the tube length of the wind instrument must be changed. The trombone accomplishes this with a slide that the player can extend or pull back in order to change the length of the tube. Other brass instruments, like the trumpet and tuba, accomplish this change in length with valves that allow the air to move through additional tubes, thereby increasing the overall length of the standing wave. Finally, the woodwinds change tube length by opening or closing tone holes along the length of the tube. An open hole on a pipe, if large enough, defines the virtual end of the tube.

MORE ABOUT BRASS INSTRUMENTS



The trumpet, trombone, and French horn are all closed pipes with long cylindrical sections and should therefore only be able to produce odd harmonics. The length of a B^b trumpet is 140 cm. A closed cylindrical pipe with the same length produces a fundamental frequency of 61 Hz. It's higher modes are odd integer multiples of this first harmonic (see Table 6.1). However, as with all brass instruments, the mouthpiece and the bell have a significant effect on the resonant frequencies. The cylindrical piece of pipe without a bell or mouthpiece will reflect all of its standing wave modes at the same point – the end of the pipe. But add a bell to the pipe and the modes will reflect at different points. The lower the frequency of the mode, the earlier it "sees" the flare of the bell. So the lower frequency modes become shorter in wavelength as a result of the bell. This shorter wavelength increases the frequencies of the lower modes (see Figure 6.11). The mouthpiece also has an effect. It is approximately 10 cm long and has its own fundamental frequency of about 850 Hz. This frequency is known as the popping frequency because of sound that "pops" from the mouthpiece if it is removed from the trumpet and struck against the hand. However, the mouthpiece retains some of its identity even when it is inserted into the trumpet. It's presence affects the frequencies of the trumpet's higher modes, decreasing their frequency and also increasing their prominence in the total spectrum of the trumpets sound. Together, the bell and mouthpiece cause the sound production of the trumpet, trombone, and French horn to be like that of an open pipe (having all harmonics instead of just the odd ones – see Table 6.1). The presence of these modified resonance modes provides greater feedback to the player and enhances his ability to "find" a particular mode. The lowest note the B^b trumpet is designed to play is B^b_3 (233 Hz). The actual "harmonic" frequencies that a high quality B^b trumpet is able to produce are shown in table 6.1.

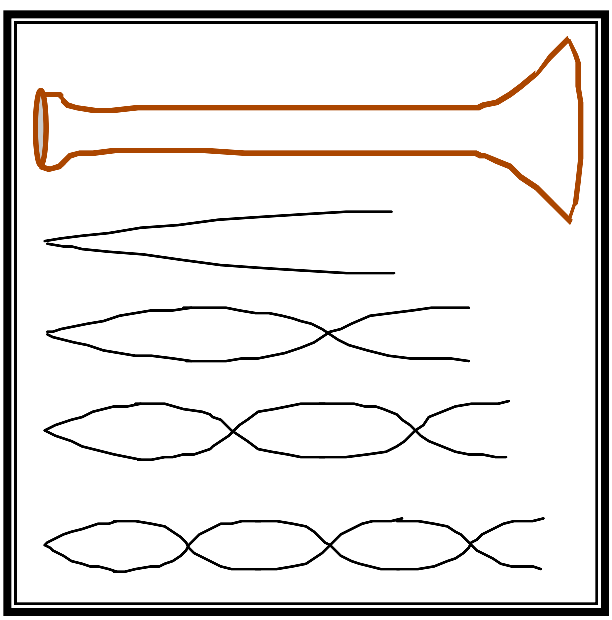


Figure 6.11: The bell of a brass instrument causes lower modes to reflect prior to reaching the end of the instrument. This smaller wavelength for the lower modes increases their frequencies, forcing them to approach the harmonicity of an open pipe.

Mode	Frequency within a closed 140 cm cylindrical pipe (Hz)	Frequency within the B^b trumpet (Hz)
1	61	not playable
2	184	230
3	306	344
4	429	458
5	551	578
6	674	695
7	796	814
8	919	931

Table 6.1: Resonant mode frequencies for a closed 140 cm cylindrical pipe vs. those obtained by the B^b trumpet, also 140 cm long, but with a bell and mouthpiece (Berg, Stork)

Clearly the frequencies are neither truly harmonic nor are they notes in the scale of equal temperament, but they are close enough that a good trumpet player can "lip up" or "lip down" the frequency with subtle lip changes as he listens to other players in a band or orchestra.

Table 6.1 shows that the interval between the two lowest notes produced by the trumpet is approximately a fifth (□B^b_3 to □F_4). That leaves six missing semitones. In order to play these missing notes, the player uses the valves on the trumpet (see Figure 6.12). When the valves are not depressed, air flows only through the main tube. However, when a valve is pressed down, the air is forced to flow through an additional tube linked to that tube (Figure 6.13). The three tubes lengthen the trumpet by an amount that changes the resonant frequency by a semitone, whole tone, or a minor third (three semitones). By using valves in combination, the trumpet can be lengthened by an amount that changes the resonant frequency by four, five, or six semitones.

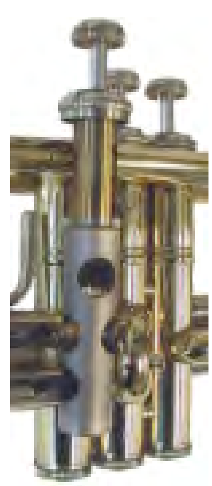


Figure 6.12: Each trumpet valve has two paths through which air can flow. When the valve is not depressed, it allows air to flow through the primary tube. When the valve is depressed, air is forced through different chambers that divert it through an additional length of tube. This additional length causes the standing sound wave to be longer and its frequency to be lower.

Figure 6.13: Pressing one or more of the trumpet's three valves provides additional tubing to lengthen the standing sound wave. This photo transformation of the trumpet (courtesy of Nick Deamer, Wright Center for Innovative Science Education) helps to visualize the role of each valve on the trumpet.

MORE ABOUT WOODWIND INSTRUMENTS

The woodwinds are so named because originally they were mostly constructed from wood or bamboo. Wood is still preferred for many modern woodwinds, however metal is used in constructing flutes and saxophones and plastic is used to make recorders. To change the pitch of the woodwind, tone holes along the side of the instrument are covered and uncovered to produce the desired pitch. The simplest way to look at the function of a tone hole is that, if it is open, it defines the new end of barrel of the instrument. So, a single pipe can actually be turned into eight different acoustic pipes by drilling seven holes along the side of the pipe. The length of any one of these eight virtual pipes would simply be the distance to the first open hole (which the wave sees as the end of the pipe). Consider making the placement of the holes so that the standing waves produced had frequencies of the major scale. If a tone were generated in the pipe with all the holes covered and then the holes were released one by one, starting with the one closest to the actual end of the pipe and working backward, the entire major scale would be heard.

It's not as easy as it sounds though. Choosing the position of a hole, as well as its size, is not as trivial as calculating the length of a pipe to produce a particular frequency and then drilling a hole at that point. Think about the impedance difference the wave in the pipe experiences. It's true that when the standing wave in the instrument encounters an open hole it experiences a change in impedance, but if the hole were a pinhole, the wave would hardly notice its presence. On the other hand, if the hole were as large as the diameter of the pipe, then the wave would reflect at the hole instead of the true end, because there would be no difference between the two and the hole would be encountered first. So the open hole only defines the new end of the pipe if the hole is about the same size as the diameter of the pipe. As the hole is drilled smaller and smaller, the virtual (or acoustic) length of the pipe approaches the actual length of the pipe (see Figure 6.14). Structurally it's unreasonable to drill the holes as large as the diameter. And if the bore of the instrument were larger than the fingers, then drilling large holes would require other engineering solutions to be able to fully plug the hole (see Figure 6.15).

It gets even more complicated. Even the presence of closed holes has an effect on the standing wave. The small amount of extra volume present in the cavity under the closed hole (due to the thickness of the pipe) causes the pipe to appear acoustically longer than the actual length of the pipe. And don't forget the end effect at that first open hole. Even the presence of the open holes past the first one have an effect. If they are spaced evenly they will tend to reflect lower frequencies more strongly than higher ones. Indeed, the presence of these open holes leads to a **cutoff frequency**. Above this critical frequency, sound waves are reflected very little, giving the woodwinds their characteristic timbre.

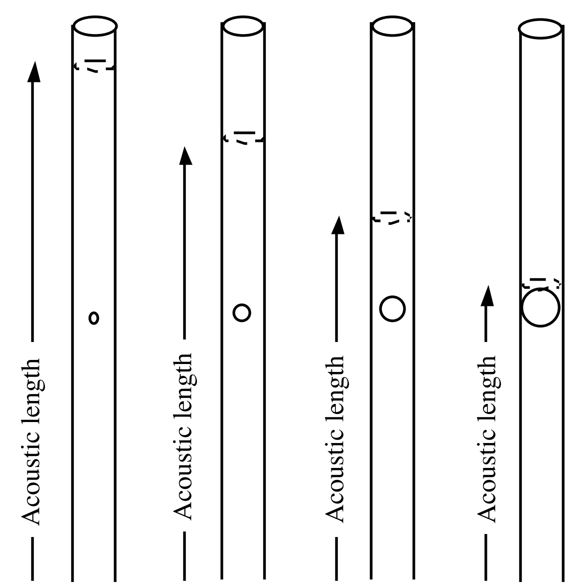


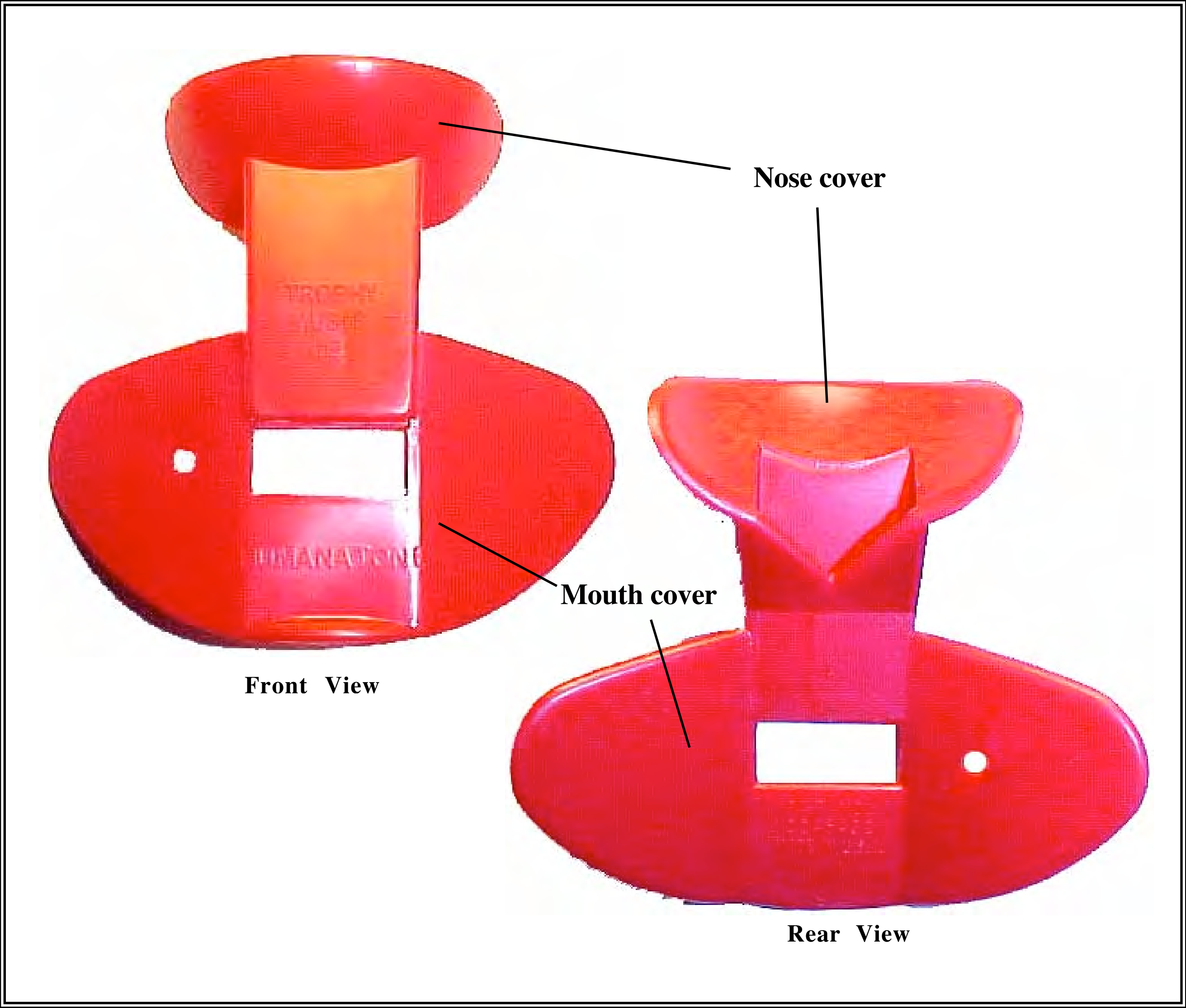
Figure 6.14: A hole drilled on the side of a pipe changes the acoustic length of the pipe. The larger the hole, the closer the acoustic length will be to the hole position.

So the question is, can the equations for the frequencies of standing waves within an open or closed pipe be used to determine the position and size of tone holes? The answer is … no, not really. The consequences of so many different factors leads to complicated equations that give results that are only approximations. Actual woodwind construction is based on historic rules of thumb and lots of trial and error.

Figure 6.15: From the recorder to the clarinet to the saxophone, tone holes go from small and simple to small and complex to large and complex. As the instrument grows in length and diameter, the tone holes get further apart and must also grow in diameter. Compare the simple tone holes of the recorder, which can be easily covered with the player's fingers to the tone holes of the saxophone, which must be covered with sophisticated multiple, large diameter hole closer systems.

INVESTIGATION

THE NOSE FLUTE



INTRODUCTION

The nose flute is a curious musical instrument. Actually it is only part of a musical instrument, the remainder being the mouth cavity of the player. To play it, the rear of the nose flute is placed over the nose and open mouth of the player and air is forced out of the nose. The nose flute directs this air across the mouth in the same way that one might direct air over the top of a soda bottle to produce a tone. The result is a clear, pleasing, flute-like tone. Click here to listen to the nose flute. These are available at a variety of sites, including one selling them for only $.99 per nose flute: http://www.funforalltoys.com/products/just_for_fun_3/nose_flute/nose_flute.html

QUESTIONS AND CALCULATIONS (MAKE CLEAR EXPLANATIONS THROUGHOUT AND SHOW ALL CALCULATIONS CLEARLY)

1. What type of musical instrument classification fits the nose flute? Be specific, indicating the vibrating medium and how the pitch of the nose flute is changed.

2. If you and another person nearby both played nose flutes, could you produce beats? How would you do it?

3. What are the similarities and differences between whistling and playing the nose flute?

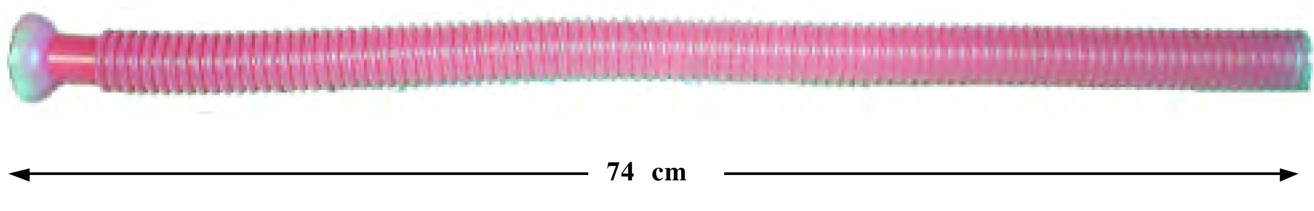
4. Is the pitch range for the nose flute fixed or different for various players? Explain.

5. If you played the nose flute, what is the lowest theoretical note on the Equal Tempered Scale that you personally would be able to get? (You will need to make a measurement to answer this question.)

6. What two things could you do to produce a tone an octave and a fifth above the lowest theoretical frequency calculated in the previous problem?

INVESTIGATION

THE SOUND PIPE



INTRODUCTION

The sound pipe is a fun musical toy. To operate the sound pipe, you simply hold one end and twirl the sound pipe in a circle. The movement of the end not being held causes a low-pressure region in the air. This is due to the Bernoulli Effect and is also the explanation for how the perfume atomizer works. In the case of the atomizer, air puffed past the tube connected to the perfume creates a low-pressure region above the tube, thus causing the perfume to rise into the path of the puffed air. In the case of the sound pipe, the low-pressure region caused by its motion draws air into the pipe. Depending on how fast it is twirled, it is possible to make four different audible frequencies. The inside diameter of the sound pipe is 2.5 cm. Both ends of the sound pipe are open.

QUESTIONS AND CALCULATIONS (SHOW ALL WORK)

1. What type of musical instrument classification fits the sound pipe? Be specific, indicating the vibrating medium.

2. What is the relationship between the four possible audible frequencies?

3. When you twirl this sound pipe, what is the lowest pitch you could produce?

4. What are the other three frequencies this sound pipe is capable of producing? What notes do the four possible frequencies represent? You can listen to all four frequencies here.

5. In order to most easily use the sound pipe to play music, you would really need several sound pipes of different lengths. Let's say you wanted to be able to play all the notes within a C major scale.
   a. First determine what length to cut from this sound pipe so that it produces the frequency of the C closest to the fundamental frequency of this sound pipe.

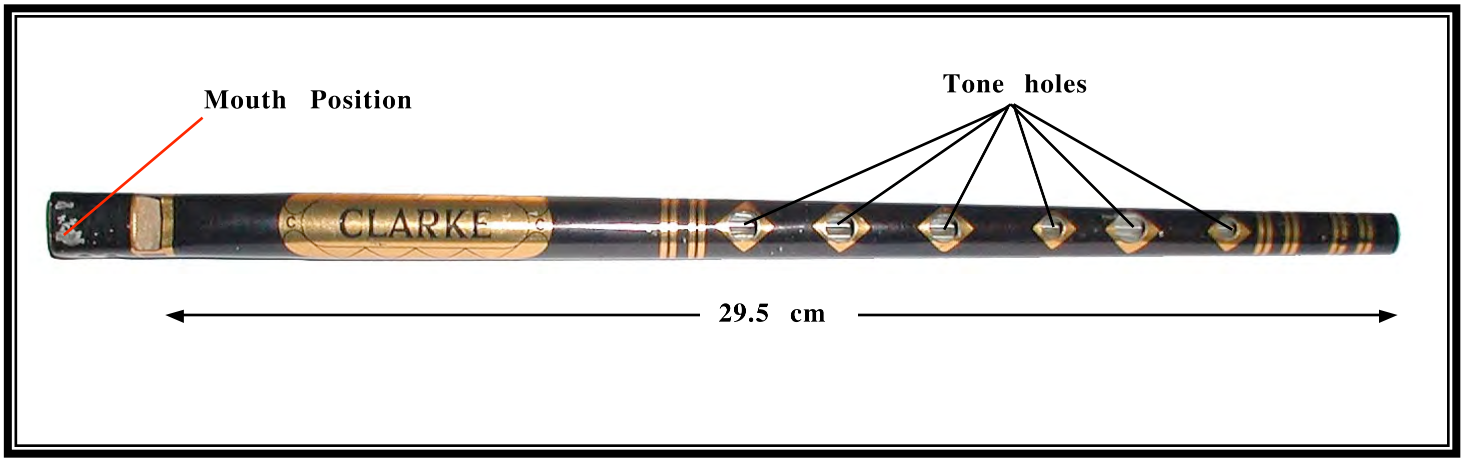
   b. Now calculate what the lengths the other six sound pipes would have to be in order to produce a full C-major scale (assume the fundamental frequency will be used for each pipe).

6. Imagine you had two sound pipes identical to the one in the photograph. If 1.0 cm were cut from one of them, what specifically would be heard if they were both twirled so that they were resonating in the second mode?

7. Another company produces a sound pipe of the same length, but only 2.0 cm in diameter. If the sound pipes from both companies were twirled so that they produce standing waves vibrating in the third mode, what specific sound would be heard?

INVESTIGATION

THE TOY FLUTE



INTRODUCTION

The inexpensive toy flute pictured here is a slightly tapered metal tube, open at both ends. There is also an opening near where the mouth is placed as well as six tone holes. The length of the flute from the opening on top near the mouth to the end of the flute is 29.5 cm. The diameter of the end of the flute is 1.0 cm and it increases to 1.4 cm at the position of the tone hole closest to the mouth. The walls of the pipe are very thin – less than 0.1 cm. The diameters of the tone holes in order, starting from the one closest to the mouth are: 0.6 cm, 0.6 cm, 0.6 cm, 0.5 cm, 0.7 cm, and 0.5 cm.

CALCULATIONS (EXPLAIN THE PROCESS AND REASONING YOU'RE USING THROUGHOUT. SHOW ALL CALCULATIONS CLEARLY)

1. a. What is the lowest note the flute is designed to play?

   b. How would this note change if the flute were played outside on a hot day with the temperature of 37°C instead of inside at room temperature?

2. How many tones could be produced with this flute if only the first mode were used? Explain.

3. Let's assume for this question that the tone holes were all the same diameter as the pipe (at the position that the tone hole is drilled in the pipe). If played only in the first mode, could the flute produce a tone an octave or more higher than the lowest possible frequency?

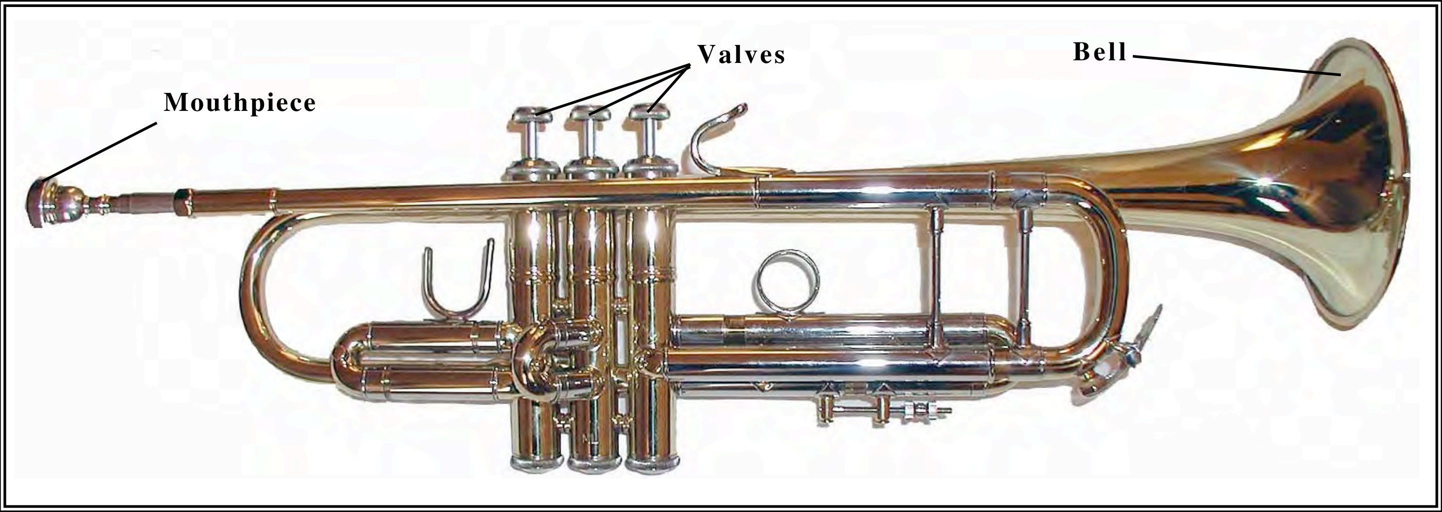
4. When this actual flute is played in the first mode, with all the tone holes uncovered, it produces a note 11 semitones higher than the note produced when all the tone holes are covered. Explain why the flute is unable to play notes an octave or more higher than the lowest possible frequency.

5. What would you have to do with this flute (without drilling anymore holes) in order to produce tones an octave or more higher than the lowest possible frequency?

6. It is possible (although it doesn't sound very pleasant) to produce the third mode with this flute. What is the highest note the flute is capable of playing?

INVESTIGATION

THE TRUMPET



INTRODUCTION

The trumpet is a closed pipe with a predominantly long cylindrical section. If it were cylindrical throughout its length it would only be able to produce odd harmonics. However, as with all brass instruments, the mouthpiece and the bell have a significant effect on the resonant frequencies. The bell increases the frequency of the lower modes and the mouthpiece decreases the frequency of the higher modes. The mouthpiece also increases the prominence of particular frequencies. Together, the bell and mouthpiece cause the sound production of the trumpet to be like that of an open pipe. The B^b trumpet is 140 cm and the lowest note it is designed to play is B^b_3 (233 Hz). The actual "harmonic" frequencies that a high quality B^b trumpet is able to produce are shown in the table below:

Mode	Freq. (Hz)
2	230
3	344
4	458
5	578
6	695
7	814
8	931

Frequencies obtained by the B^b trumpet (Berg, Stork)

Clearly the frequencies are not truly harmonic nor are they notes in the Equal Temperament Scale, but they are close enough that a good trumpet player can "fine tune" the frequency with subtle lip changes as he listens to other players in a band or orchestra.

The table shows rather large intervals (the interval between the second and third harmonic is about a fifth). In order to play smaller intervals, the player uses the valves on the trumpet. When the valves are not depressed, air flows through them in the main tube. However, when a valve is pressed down, the air is forced to flow through an additional tube section. The three tubes lengthen the trumpet by an amount that changes the resonant frequency as follows:

- Valve 1 (valve closest to player's mouth) – 1 whole tone.
- Valve 2 – 1 semitone
- Valve 3 – minor third (three semitones)

By using valves in combination, the trumpet can be lengthened by an amount that changes the resonant frequency by four, five, or six semitones.

CALCULATIONS (EXPLAIN THE PROCESS AND REASONING YOU'RE USING THROUGHOUT. SHOW ALL CALCULATIONS CLEARLY)

1. If the bell and mouthpiece of the trumpet were not present and the trumpet was still 140 cm long, what would be the frequencies and approximate notes of the first four modes?

2. There is a frequency, known as the pedal tone that is not normally played on the trumpet. It is the fundamental frequency (first harmonic) for the trumpet. What is this frequency?

3. Use the table on the previous page to discuss, as quantitatively as you can, how closely the modes of the trumpet are to being harmonic.

4. With the bell and mouthpiece in place, what is the effective length of the trumpet?

5. Recall that the ratio of frequencies between notes that are separated by one semitone is 1.05946. So from semitone to semitone, the frequencies increase or decrease by 5.946%. The ratios between lengths of pipes that play consecutive semitones have a similar relationship. Calculate the lengths of each of the three tubes that can be activated by the valves of the trumpet.

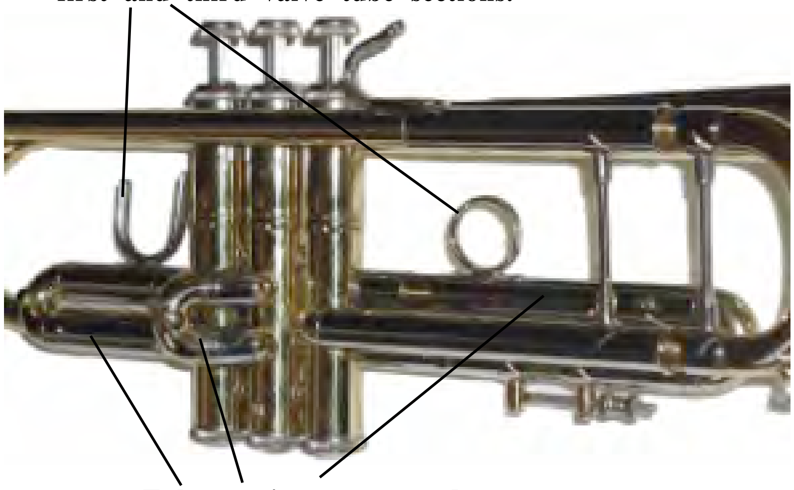
6. The trumpet valves can be used in combination to change the resonant frequencies by more than three semitones. Use the results from the previous problem to determine the extra lengths of tubing in the trumpet when multiple valves are pressed at the same time to lower the resonant frequency by four, five, or six semitones.

7. Now assume that the trumpet actually had six valves and the trumpet could be lengthened by an amount that changed the resonant frequency by one two, three, four, five, or six semitones. Calculate the length of individual tubes that would change the resonant frequencies by four, five, and six semitones.

8. Compare the extra tube lengths (producing true semitone intervals) that you calculated in the last problem with the sums of tube lengths that the trumpet actually uses to make four, five, and six semitone interval adjustments.
   a. Why is there a discrepancy?

   b. Without making any adjustments, would the notes produced by the trumpet sound flat or would they sound sharp when making four, five, and six semitone interval adjustments? Why?

9. The photograph to the right shows the three valves, the extra tube sections connected to each of them, and two adjusters for the lengths of the tube sections connected to the first and third valves. Why would there be a need for these adjusters?



10. Indicate which mode is played and which valve(s) would be pressed to produce each of the following notes.
    a. G_4

    b. F_3

    c. E_5

    d. C_4

11. The brass pipe pictured to the right has few of the trumpet's attributes. There is no mouthpiece, no bell, and no valves, but it is a brass tube that you could buzz your lips into. What are the four lowest notes that would be possible to blow on this pipe?

12. Now assume that the pipe is fitted with a bell and transformed into the horn pictured below.
    a. What would happen to each of the frequencies of the notes calculated in the previous problem?

b. What other change in the sound production of the horn would occur?

BUILDING A SET OF PVC PANPIPES

OBJECTIVE:

To design and build a set of panpipes based on the physics of musical scales and the physics of the vibrating air columns.

MATERIALS:

- One 10-foot length of 1/2” PVC pipe
- Small saw or other PVC cutter
- Fine sand paper
- Eight rubber or cork stoppers
- Approximately 1 meter of thin, 1-inch wide wood trim
- Strong, wide, and clear strapping tape
- Metric ruler or tape

PROCEDURE

1. Decide which notes the panpipes will play. The panpipes pictured to the right consist of the C_4 major scale plus a C_5 note (C_4, D_4, E_4, F_4, G_4, A_4, B_4, C_5) on the Equal Tempered scale.

2. Decide which scale you will use and then calculate the frequencies your panpipe will have. <u>Do all calculations and check work before making any cuts</u>.

3. Calculate the length of each of your pipes to the nearest millimeter, assuming the panpipes will be used in the first mode. Keep in mind the *end effect* as well as the fact that the stoppers will not only block the bottoms of the pipes, but stick up inside of them a bit too. Before making your length calculations you should check how far the stoppers enter the pipe when they are snuggly in place.

4. Carefully cut each of the pipes. Make sure that each pipe is cut precisely to the number of millimeters calculated. Mistakes in this part of the procedure will be audibly detectable and may not be correctable.

5. Sand the rough edges of the pipes caused by cutting them.

6. Insert a stopper into one end of each of the pipes. Gently blow over the top of each pipe, listening for those that may be too flat or too sharp. A stopper may be moved a small distance in or out of a pipe in order to tune it slightly.

7. Lay the pipes side by side, with their tops all flush. Measure the distance across the set of pipes. Cut four pieces of wood trim to this length. Sand the edges of the trim.

8. Place two pieces of cut trim on each side of the set of pipes; one pair 3-4 cm below the tops of the pipes and another pair 3-4 cm above the bottom of the shortest pipe. Wrap strapping tape tightly (several times) around each pair of trim strips. Wrap one piece of tape around the panpipe between the two pairs of wood trim.

9. Play the panpipes! You can play tunes and experiment by listening to octaves, fifths, fourths, and other intervals.

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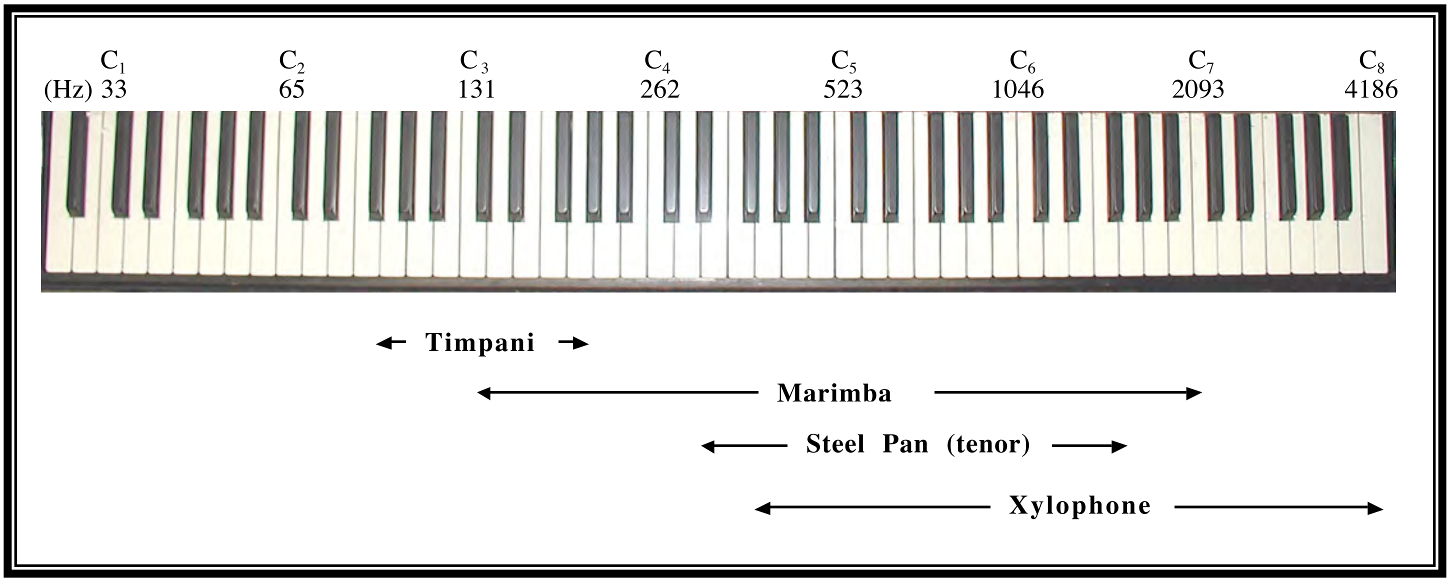
"Music should strike fire from the heart of man, and bring tears from the eyes of woman."
– Ludwig Von Beethoven

CHAPTER 7 IDIOPHONES (PERCUSSION INSTRUMENTS)



IT'S PRETTY HARD to pass by a set of wind chimes in a store and not give them a little tap. And few of us leave childhood without getting a child's xylophone for a gift. The sounds produced when pipes or bars are tapped on their sides are fundamentally different from the sounds produced by the instruments in the previous two categories. That's because the frequencies of higher modes in vibrating pipes and bars are not harmonic. Musical instruments consisting of vibrating pipes or bars are known as idiophones.

BARS OR PIPES WITH BOTH ENDS FREE

In a bar whose ends are free to vibrate, a standing wave condition is created when it is struck on its side, like in the case of the marimba or the glockenspiel. **The constraint for this type of vibration is that both ends of the bar must be antinodes**. The simplest way a bar can vibrate with this constraint is to have antinodes at both ends and another at its center. The nodes occur at 0.224 L and 0.776 L. This produces the fundamental frequency (see Figure 7.1).

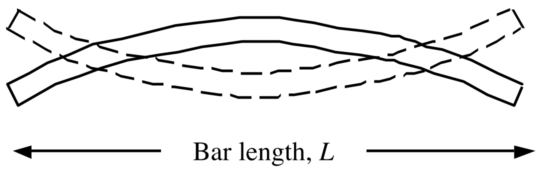


Figure 7.1: First mode of vibration. This is the simplest way for a bar or pipe to vibrate transversely in a standing wave condition with both ends free. This mode generates the fundamental frequency.

The mode of vibration, producing the next higher frequency, is the one with four antinodes including the ones at both ends. This second mode has a node in the center and two other nodes at 0.132 L and 0.868 L (see Figure 7.2).

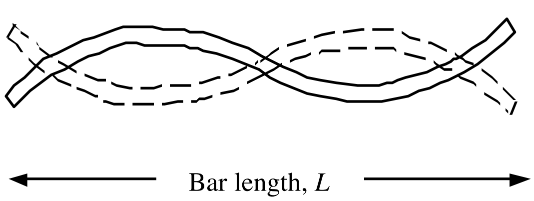


Figure 7.2: Second mode of vibration. This is the next simplest way for a bar or pipe to vibrate in a standing wave condition with both ends free. This mode generates the first overtone.

The mathematics used to describe this particular vibration of the bar is pretty complicated, so I'll just present the result. (Fletcher and Rossing in The Physics of Musical Instruments, Vol. 2, pp 56 – 64 give a full mathematical development). If the bar is struck on its side, so that its vibration is like that shown, the frequency of the n^{th} mode of vibration will be:

$$f_n = \frac{\pi v K}{8L^2} m^2$$

Where: v = the speed of sound in the material of the bar (Some speeds for common materials are shown in Table 7.1.)

Material	Speed of sound, v (m/s)
Pine wood	3300
Brass	3500
Copper	3650
Oak wood	3850
Iron	4500
Glass	5000
Aluminum	5100
Steel	5250

Table 7.1: Speed of sound for sound waves in various materials (Askill)

L = the length of the bar

m = 3.0112 when n = 1, 5 when n = 2, 7 when n = 3, … (2n + 1)

$$K = \frac{thickness\ \ of\ \ bar}{3.46}$$ for rectangular bars

or

$$K = \frac{\sqrt{(inner\ \ radius)^2 + (outer\ \ radius)^2}}{2}$$

for tubes

Do you get it? (7.1)

a. In the space below, draw the third mode of vibration for a copper tube (both ends free), with an outer diameter of 2.5 cm, an inner diameter of 2.3 cm and a length of 50 cm.

b. Now calculate the frequency of the second mode for this bar.

BARS OR PIPES WITH ONE END FREE

Another type of vibration for bars is when one of the ends is clamped, like in a thumb piano. The free end is struck or plucked, leading to a standing wave condition in which **the constraint is that the clamped end is always a node and the free end is always an antinode**. The simplest way the bar can vibrate is with no additional nodes or antinodes beyond the constraint. This produces the fundamental frequency (see Figure 7.3).

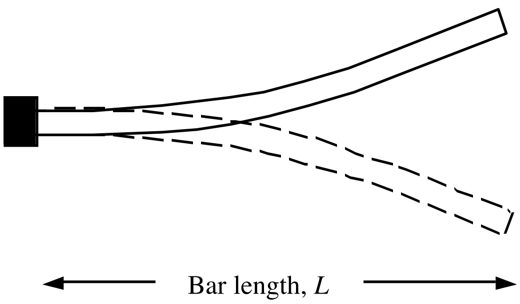


Figure 7.3: First mode of vibration. This is the simplest way for a bar or pipe to vibrate transversely in a standing wave condition with only one end free. This mode generates the fundamental frequency.

The next mode of vibration, producing the next higher frequency, is the one with two antinodes and two nodes including the node and antinode at each end (see Figure 7.4).

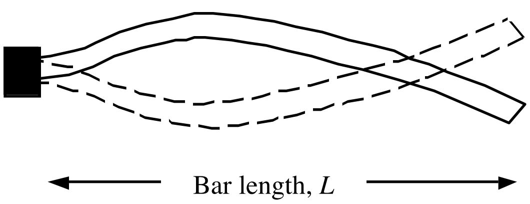


Figure 7.4: Second mode of vibration. This is the next simplest way for a bar or pipe to vibrate in a standing wave condition with both ends free. This mode generates the first overtone.

The expression for the n^{th} frequency of the clamped bar looks identical to that of the bar with free ends. The only difference is in the value of "m". If the bar is plucked or struck on its side, so that its vibration is like that shown, the frequency of the n^{th} mode of vibration will be:

$$f_n = \frac{\pi v K}{8L^2} m^2$$

Where: $m =$ 1.194 when n = 1, 2.988 when n = 2, 5 when n = 3, … (2n - 1)

And all other variables are defined identically to those of the bar with free ends equation.

Do you get it? (7.2)

a. In the space below, draw the third mode of vibration for an aluminum bar, with a thickness of 0.75 cm and a length of 50 cm.

b. Now calculate the frequency of the second mode for this bar.

As mentioned earlier, the frequencies of the modes of transversely vibrating bars and pipes are different from those of vibrating strings and air columns in that they are not harmonic. This becomes obvious when looking at the last two equations for transverse vibration frequency of bars and pipes. In both cases, $f_n \propto m^2$, where f_n is the frequency of the n^{th} mode and m is related to the specific mode. For transversely vibrating bars and pipes with free ends:

$$\frac{f_2}{f_1} = \frac{5^2}{3.0112^2} = 2.76 \text{ and } \frac{f_3}{f_1} = \frac{7^2}{3.0112^2} = 5.40.$$

Do you get it? (7.3)

Use the space below, to calculate the relationships between the frequencies of different modes of transversely vibrating pipes and bars. Complete the table below with your results.

Both Ends Free		**One End Free**	
Mode	Multiple of f_1	Mode	Multiple of f_1
1	1	1	1
2	2.76	2	
3	5.40	3	
4		4	
5		5	

TOWARD A "HARMONIC" IDIOPHONE

It was shown earlier that a transversely vibrating bar with both ends free to move has a second mode vibration frequency 2.76 times greater than that of the first mode. The third mode has a frequency 5.40 times greater than that of the first mode. These are obviously not harmonic overtones. Recall, that the interval for one semitone on the 12-tone Equal Tempered scale is 1.05946. And $(1.05946)^{12} = 2$. This relationship can be used to find the number of 12-tone Equal Tempered semitones that separate the modes of the transversely vibrating bar:

$$\log(1.05946)^{12} = \log 2 \;\Rightarrow\; 12\log(1.05946) = \log 2$$
$$\Rightarrow\; \frac{\log 2}{\log(1.05946)} = 12.$$

Or more generally, the number of 12-tone Equal Tempered semitones for any interval is equal to:

$$\textit{Equal Tempered semitones} = \frac{\log(\textit{interval})}{\log(1.05946)}$$

So for the transversely vibrating bar the interval between the first and second mode is:

$$\frac{\log(2.76)}{\log(1.05946)} = 17.6 \;\; \textit{semitones}.$$

And the interval between the first and third mode is:

$$\frac{\log(5.40)}{\log(1.05946)} = 29.2 \;\; \textit{semitones}.$$

These clearly do not match the 12 semitones of the octave, the 24 semitones of two octaves or the 36 semitones of three octaves, but there are other combinations of consonant intervals that could be considered. 19 semitones would be equivalent to an octave plus a fifth and 17 semitones would be equivalent to an octave plus a fourth, either of which would be consonant. And 29 semitones would be equivalent to two octaves plus a fourth. This is pretty close to the 29.2 semitones of the third mode, but it's really a moot point because the third mode ends up dying out so quickly anyway. The real concern is for the second mode which is much more persistent. The second mode is not only inharmonic; it isn't even musically useful as a combination of consonant intervals. This causes unmodified idiophones to have less of a clearly defined pitch than harmonic instruments. However, a simple modification can be made to the bars of xylophones and marimbas to make the second mode harmonic.

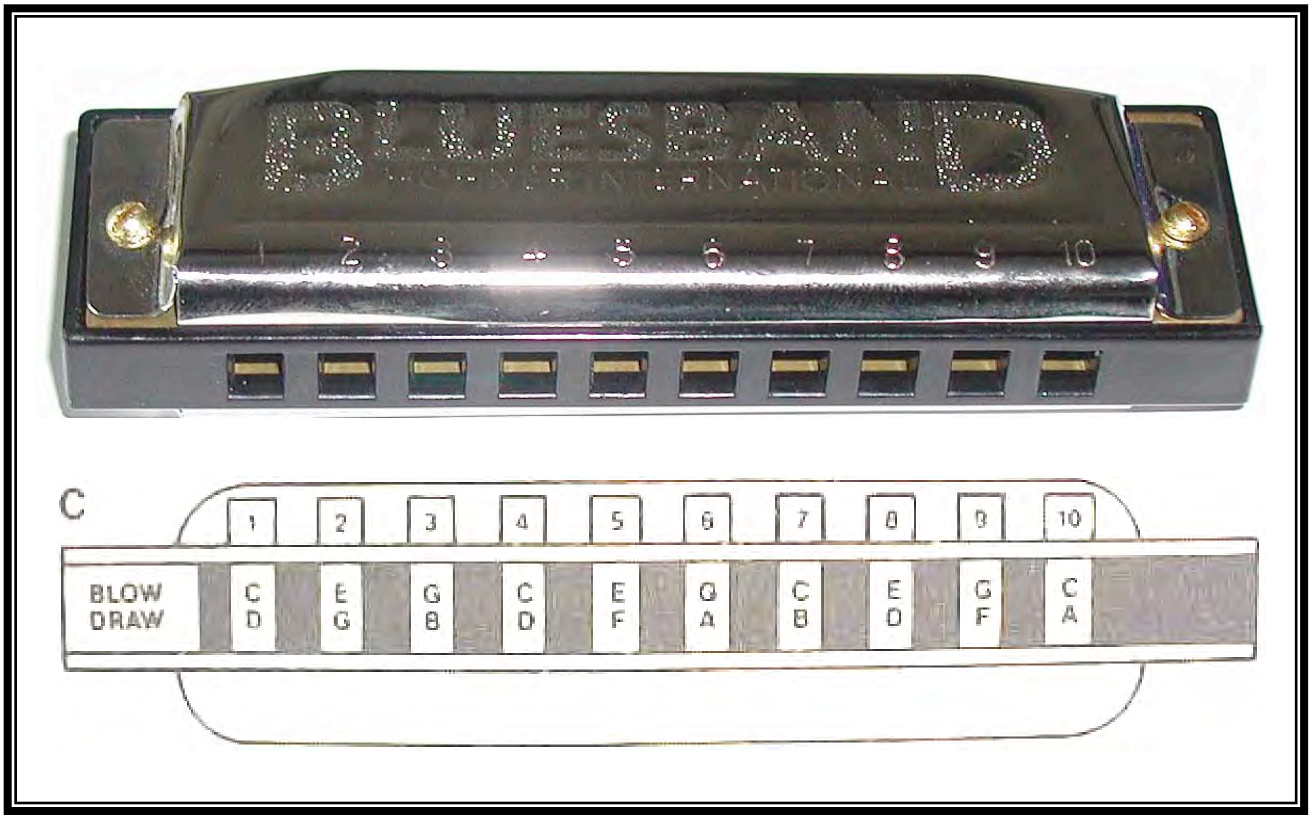
Figures 7.1 and 7.2 show that the center of the transversely vibrating bar is an antinode in the first mode and a node in the second mode. Carving out some of the center of the bar makes it less stiff and decreases the frequency of the first mode. However, it has little effect on the second mode, which bends the parts of the bar away from the center. An experienced marimba builder can carve just the right amount of wood from under the bars so that the first mode decreases to one-quarter of the frequency of the second mode (see Figure 7.5). The xylophone maker carves away less wood, reducing the frequency of the first mode to one-third the frequency of the second mode. Both modifications give the instruments tones that are clearly defined, but the two octave difference between the first two modes on the marimba gives it a noticeably different tone than the xylophone's octave-plus-a-fifth difference between modes.

Figure 7.5: The second mode of xylophone and marimba bars is made harmonic by carving wood from the bottom center of the bar. This lowers the fundamental frequency of the marimba bars to one-quarter the frequency of the second mode and lowers the fundamental frequency of the xylophone bars to one-third the frequency of the second mode.

View a 20 minute movie about the Javanese Gamelan. Click the box above.

INVESTIGATION

THE HARMONICA



INTRODUCTION

Many people may consider the harmonica to be a wind instrument. But the harmonica makes all its sound by means of reeds, bound at one end, that are driven to vibrate by the breath of the player. There are no resonant tubes or pipes. In the photograph above you can see ten holes in the front of the harmonica through which you can either blow air into or draw air from.

The harmonica's history can be traced back to a musical instrument called the "aura." In 1821, 16-year-old Christian Friedrich Buschmann registered the aura for a patent. His instrument consisted of steel reeds that could vibrate alongside each other in little channels. Like the modern design, it had blow notes. However, it had no draw notes. Around 1826 a musical instrument maker named Richter changed the design to its modern style – ten holes with 20 reeds, and tuned to the diatonic scale (Buschmann's design used the chromatic scale).

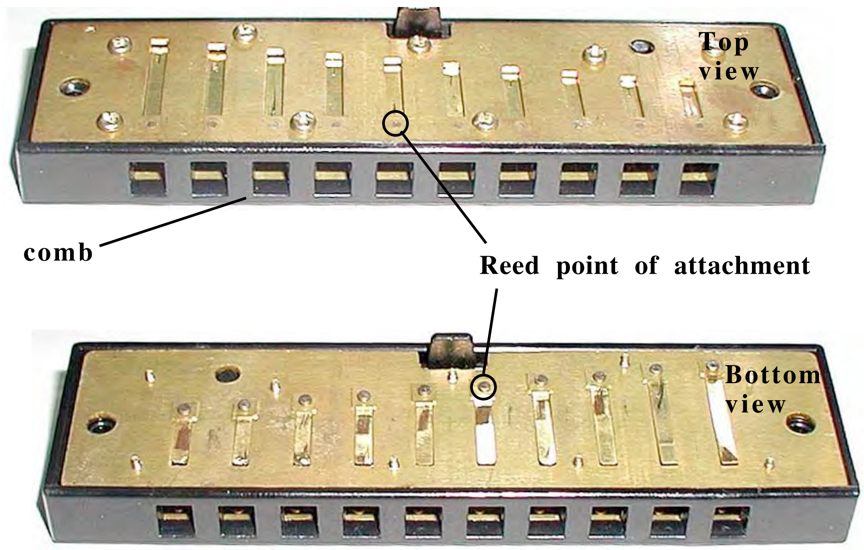
For as little as $2.00 apiece educators may purchase inexpensive harmonicas from Hohner, Inc.

Hohner Contact:

Johnna Cossaboon
Marketing Communications Manager
Hohner, Inc./HSS
1000 Technology Park Drive
Glen Allen, VA 23059
804-515-1900 ext. 3043
804-515-0840 FAX

QUESTIONS AND CALCULATIONS

1. If the chrome cover is taken off the harmonica, the reed plates can be seen, joined to the comb. The top photograph shows the top of the harmonica. Notice that you can see the channels in which the blow reeds vibrate. The reeds are attached to the underside of the metal plate and if you look carefully, you can see the point of attachment. The free end of the blow reed is the end farthest from where the mouth blows. If the harmonica is now turned over, it appears as the bottom photograph and you can see the draw reeds above the channels in which they vibrate. You can also see the points of attachment. The free end of the draw reed is the end closest to where the mouth blows. Plucking the reeds produces a sound that is hardly audible.

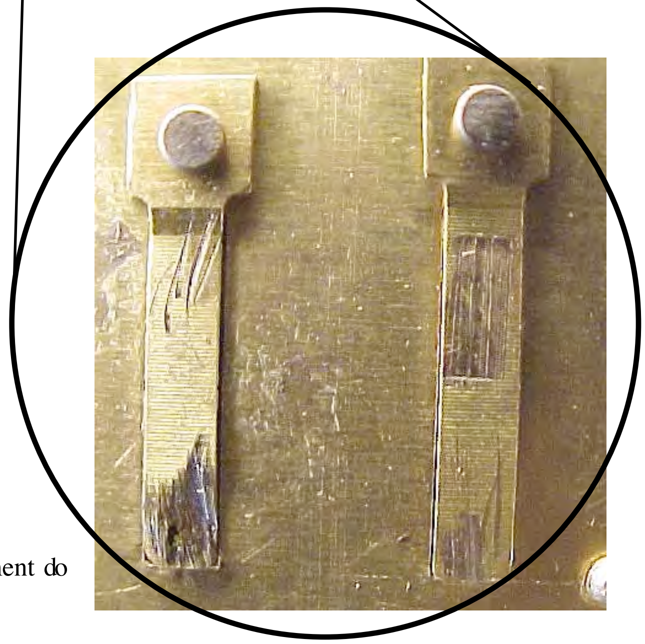


   a. Use the photographs to explain how the reeds are driven to vibrate and what the mechanism is for the production of sound from the harmonica?

   b. Why must the reeds be mounted on the particular sides of the reed plates that they are attached to?

2. A close look at the draw reeds shows that they all have evidence of being filed (the blow reeds do too). Some of the reeds are filed up near the point of attachment and others have been filed at their free ends. Some reeds are filed in both regions. The file marks have the appearance of being random, but they are actually intentional. Filing the end of a reed causes it to behave like a shorter reed. Filing the reed near the point of attachment causes it to behave like a thinner reed.

   a. What does filing the end of a reed do to the frequency of the reed when it is vibrating?



   b. What does filing the reed near the point of attachment do to the frequency of the reed when it is vibrating?

   c. In the magnified section, you can see that the reed on the left has been filed at both the end and near the point of attachment. The end shows a greater degree of filing. Propose a scenario to explain why both parts of the reed would be filed and why the end would have a greater degree of filing done on it.

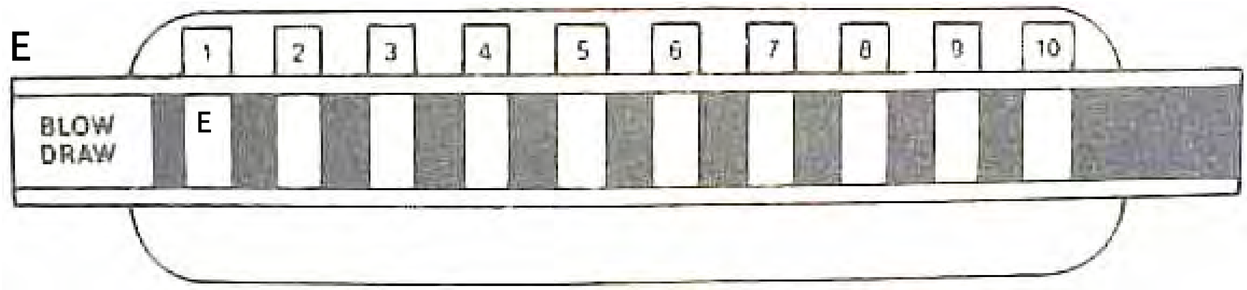
3. The lowest note on this harmonica is C_4. The length of this reed is 1.60 cm. How long would a reed identical to those in this harmonica need to be to play C_3?

4. The blow and draw notes for the C major harmonica are shown on the photograph on the first page of this activity. There are three ways to look at the organization of notes on the harmonica. One way is as a system of chords (two or more consonant notes played at the same time). A second way to look at the note structure is by thinking of it as based around the major scale. Finally, the harmonica can be thought of in terms of an abundance of octaves. Explain how each of these viewpoints is valid. Remember that you can get your mouth around multiple holes and you can also use your tongue to block air from moving through unwanted channels.
   a. A system of chords

   b. The major scale

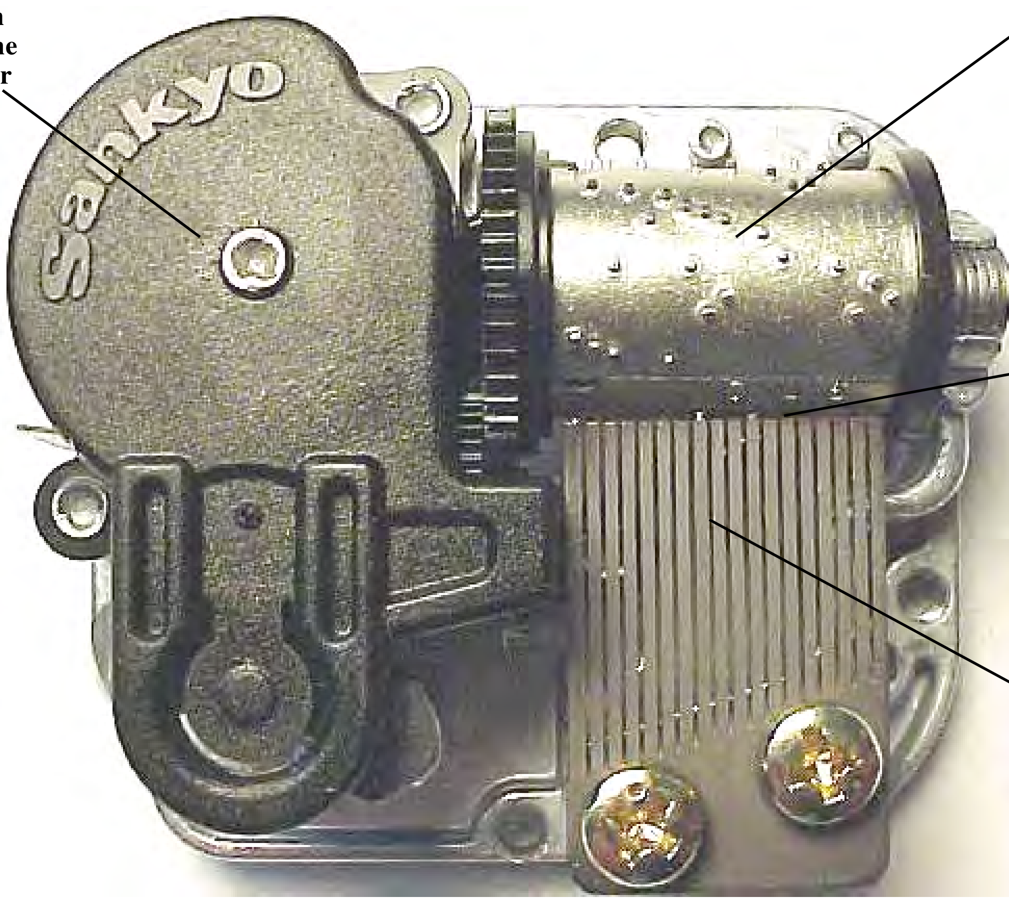
   c. An abundance of octaves

5. The figure below shows an E major harmonica. The first draw hole note is an *E*. Use your answers from the previous question to decide on and fill in the other blow holes and all the draw holes. Use the space below the figure to fully explain your rationale.



INVESTIGATION

THE MUSIC BOX ACTION



INTRODUCTION

The heart of any music box is the *action*. One part of the action consists of a small metal cylinder with tiny metal projections attached to it. The other part of the action is a flat piece of metal with a set of thin metal tines, each of which can be plucked by the metal projections on the cylinder as it turns. In most cases the drum is attached to a coiled spring, which can be wound up to provide energy for the drum to rotate for a few minutes. When the action is firmly attached to the music box and wound up, it will play a clearly audible and recurring tune during the time the drum is in motion. Click here to hear this music box action. An inexpensive action can be purchased for $3.75 or only $1.85 each when ordering 50 or more at:
http://www.klockit.com/product.asp?sku=GGGKK&id=0309301913064849755057

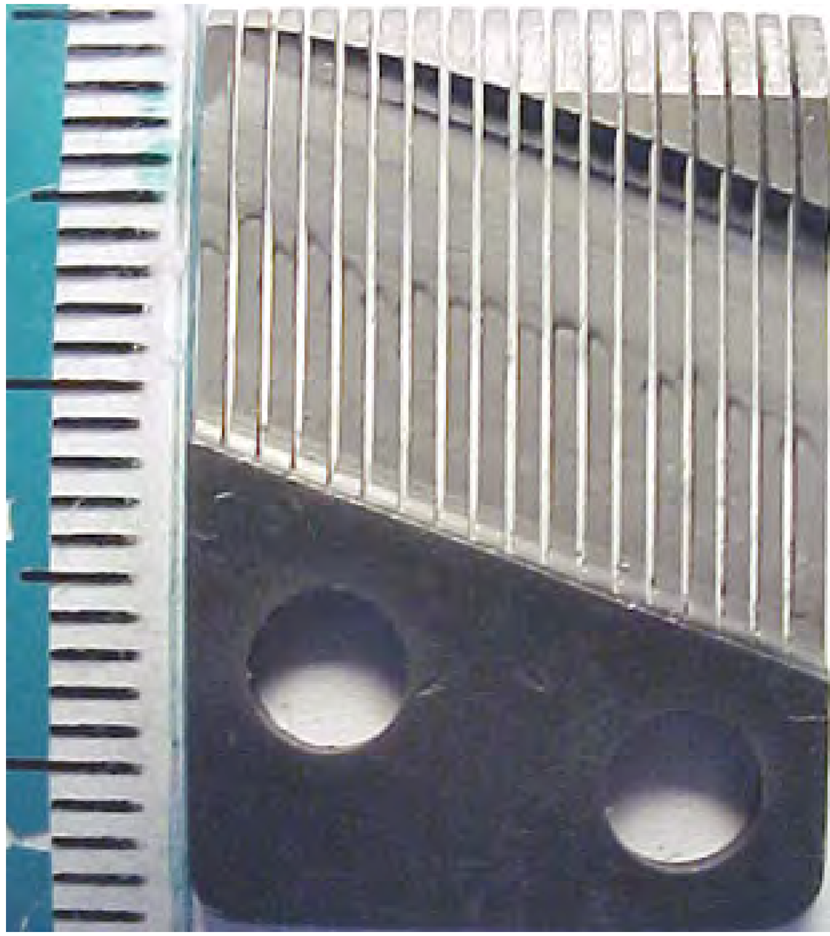
QUESTIONS AND CALCULATIONS (SHOW ALL WORK)

1. What type of musical instrument classification fits the music box action? Be specific, indicating the vibrating medium.

2. What is the means for changing the pitch of the music box?

3. How many different tones are possible with this particular music box action? Explain.

4. Each tine actually creates a pitch one semitone different from its adjacent tines. The longest of the tines creates a C_5 pitch. If the tines were all uniformly thick, what percentage of the length of the longest tine would the shortest tine be?



5. The photograph to the right shows the underside of the metal plate holding the tines. The measurement increments are millimeters. How does the percentage of the length of the shortest tine compare to your calculation in the previous problem.

6. Explain the reason for the discrepancy.

For the last two problems, assume the metal is steel and the tines are uniformly thick.

7. What would be the thickness of the longest tine?

8. What note would the shortest tine actually play?

BUILDING A COPPER PIPE XYLOPHONE

OBJECTIVE:

To design and build a xylophone-like musical instrument based on the physics of musical scales and the physics of the transverse vibrations of bars and pipes.

MATERIALS:

- One 10-foot long piece of 1/2” copper pipe
- Pipe cutter
- Rubber foam self stick weather seal
- Metric ruler or tape

PROCEDURE

The 10-foot piece of copper is long enough to make a full C_5 (or higher) major scale, plus one pipe an octave higher than the lowest note. There will still be pipe left over to use as a mallet. To determine the lengths of the pipes, you will use the equation for the frequency of a pipe with transverse vibrations: $f_n = \frac{\pi v_L K}{8L^2} m^2$

This “xylophone,” made from a single 10-foot length of 1/2” copper pipe plays the C_5 major scale.

1. Measure the inner and outer diameter of the pipe to the nearest half-millimeter. Use the corresponding radii to calculate the radius of gyration (*K*)

2. Decide what notes the xylophone will play. The xylophone pictured above consists of the C_5 major scale plus a C_6 note (C_5, D_5, E_5, F_5, G_5, A_5, B_5, C_6) on the Equal Tempered scale. If a full chromatic scale or more than one octave is desired a higher scale must be chosen.

3. Calculate the frequencies you will use. Do all calculations and check work before making any cuts.

4. The speed of sound in copper is 3650 m/s. Calculate the length of each of your pipes to the nearest millimeter, assuming the first mode of vibration.

5. Carefully cut each of the pipes. Make sure that each pipe is cut precisely to the number of millimeters calculated. Mistakes in this part of the procedure are not correctable and will be audibly detectable.

6. Measure in 22.4% of its length from both ends of each pipe and place a mark. These are the positions of the nodes in the standing wave for the first mode of vibration.

7. Cut two 3-cm pieces of the foam weather strip for each pipe and attach at the nodes of the pipes. Place each pipe on a surface so that it rests on the weather seal.

8. Cut two 20-cm pieces of copper pipe from what is left of the 10-foot piece. These will be the mallets for the xylophone. Wrap one end of each mallet several times with office tape. The more tape used, the mellower the tone.

9. Play the xylophone! You can play tunes and experiment by listening to octaves, fifths, fourths, and other intervals.

REFERENCES

AUTHOR	TITLE	PUBLISHER	COPY-RIGHT	COMMENTS
Askill, John	*Physics of Musical Sounds*	D. Van Nostrand Company, Inc., New York	1979	Chapter 5 has the nicest inductive approach to scale building of the bunch. Sound spectra provided for dozens of instruments. Lots of good practice questions at the ends of the chapters.
Backus, John	*The Acoustical Foundations of Music*	W. W. Norton & Company, Inc., New York	1969	This is one of the "classics" on musical acoustics. It is comprehensive, but very readable. Backus gives considerable detail to the sound production of many instruments.
Benade, Arthur H	*Fundamentals of Musical Acoustics*	Oxford University Press, New York	1976	This is the most classic book on musical acoustics. Chapters 17 – 24 give an exhaustive, yet very readable treatment of many musical instruments. Benade's experimental results are widely used as references in other books, but the treatment is more complete here.
Berg, Richard E Stork, David G	*The Physics of Sound*	Prentice Hall, Inc., NJ	1982	Chapter 9 covers musical scale development very completely. Chapters 10 – 14 also do a very complete conceptual coverage of various classes of musical instruments.
Fletcher, Neville H Rossing Thomas D	*The Physics of Musical Instruments*	Springer-Verlag, New York	1998	Especially technical treatment from first principles of most classes of musical instrument. Its mathematical rigor makes it unsuitable for those without an extensive and strong background in calculus.
Gibson, George N Johnston, Ian D	"New Themes and Audiences for the Physics of Music"	*Physics Today*	January 2002	A seven-page article discussing the development of a university musical acoustics course for non-science majors.
Hall, Donald E	*Musical Acoustics (third edition)*	Brooks/Cole, Pacific Grove, CA	2002	A very thorough modern textbook with an excellent breadth of and depth into musical topics. End of chapter exercises and projects are perfect for generating ideas. Highly recommended.
Hopkin, Bart	*Musical Instrument Design*	See Sharp Press, Tucson, AZ	1996	An excellent resource for those interested in building a wide range of musical instruments with lots of "tricks of the trade." It is not just a "cookbook" style of construction, but explains the physics as well.
Hopkin, Bart	*Making Simple Musical Instruments*	Lark Books, Asheville, NC	1995	Great illustrations and photographs complement this collection of about 20 very eclectic instruments. Step-by-step instructions, as well as difficulty level, accompany each instrument.

Hutchins, Carleen M (provides introduction)	*The Physics of Music: Readings from Scientific American*	W. H. Freeman and Company, San Francisco	1978	Five slim chapters on different classes of instruments that are both conceptually and technically very strong and appropriate for the high school level.
Johnston, Ian	*Measured Tones: The Interplay of Physics and Music*	Institute of Physics Publishing, London	1989	Pretty standard content for a book on musical acoustics, but not as thematic and predictable as other similar works. Johnston gives considerably more attention to the historical development of the physics of music.
Moravcsik, Michael J	*Musical Sound: An Introduction to the Physics of Music*	Paragon House Publishers, New York	1987	This book assumes the reader has NO knowledge of music, physics, or any math higher than simple computation. It is a great comprehensive reference for the non-science reader.
Neuwirth, Erich	*Musical Temperaments*	Springer-Verlag/Wien, New York	1997	A thin volume (70 pages) that provides a very complete coverage of the characteristics of musical scales, but nothing else. Comes with a CD-ROM (PC only) containing sound files.
Rossing, Thomas D Moore, Richard F Wheeler, Paul A	*The Science of Sound (third edition)*	Addison Wesley, Reading, MA	2002	This is an exhaustive "must have" resource. *The Science of Music and Musical Instruments* would describe the book better. Well suited for high school through graduate school. Highly recommended
Sethares, William A	*Tuning, Timbre, Spectrum, Scale*	Springer-Verlag, London	1997	Chapter 3 (Musical Scales) does an excellent and mathematically exhaustive treatment of various scales. The rest is beyond the scope of the high school student.
Taylor, Charles	*Exploring Music: The Science and Technology of Tones and Tunes*	Institute of Physics Publishing Inc., Philadelphia	1992	Excellent explanation of the shortcomings of various scales. Excellent survey of the physics of many instruments.
Waring, Dennis	*Making Wood Folk Instruments*	Sterling Publishing Co., Inc., New York	1990	Chapters for simple stringed, wind, and percussion instruments as well as a chapter for more complex stringed instruments. Lacking physical explanations, it is fairly "cookbook" in style. A good source for ideas though.
White, Harvey E White, Donald H	*Physics and Music: The Science of Musical Sound*	Saunders College/ Holt, Rinehart and Winston, Philadelphia	1980	This is really the Physics of Waves with a strong emphasis on Music. Good, comprehensive coverage, but the Doppler Effect and Interference seem a bit out of place here.
Yost, William A	*Fundamentals of Hearing*	Academic Press, San Diego	2000	Technical and exhaustive treatment of all aspects of hearing. Teachers may be most interested in the physiology of the ear, which is treated here in much greater depth than in a typical acoustics book

PHYSICS OF MUSIC AND MUSICAL INSTRUMENTS WEBSITES

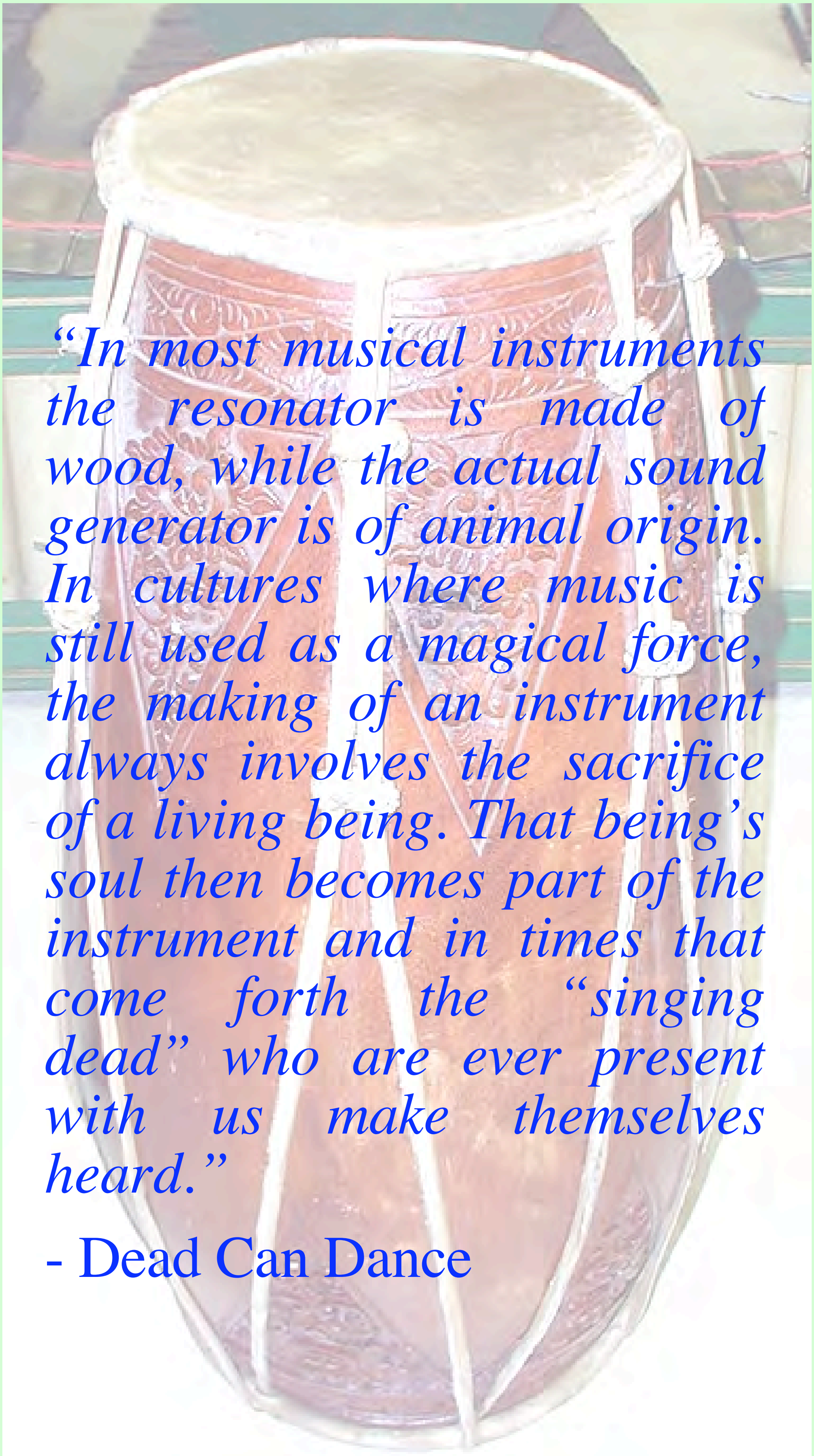
Website Address	Description	Comments
http://asa.aip.org/discs.html	Auditory Demonstrations compact Disc	This is the least expensive ($26) place to find this CD-ROM of very useful auditory demonstrations. It includes 39 demonstrations and is produced by the Acoustical Society of America.
http://hyperphysics.phy-astr.gsu.edu/hbase/sound/soucon.html#c1	The sound and hearing portion of Georgia State University's *Hyperphysics* project	Nice tutorials on many topics within sound and hearing, including much on the physics of music. Concise, navigable, and some interactivity. Students would benefit from exploring this site.
http://www.angelfire.com/tx/myquill/Harmonica.html	Diatonic Harmonica Reference	A comprehensive treatment of the harmonica
http://www.bashthetrash.com	Musical instrument construction from recycled/reused materials	This is a non-technical, but useful site to get some ideas for building different types of musical instruments from very inexpensive items (many you would normally throw away) around most any house.
http://www.carolenoakes.co.uk/InstrumentLibrary.htm	Photographs of musical instruments	There are many high quality photographs here of musical instruments by their class. While there are no descriptions, all images are clickable to reveal larger versions
http://www.exhibits.pacsci.org/music/Instruments.html	Musical instrument families	This is a nice glossary of information about the various musical instrument families that includes the following sections for each family: • How the sounds are made • How the pitch is changed • Examples of instruments in the family
http://www.klockit.com/product.asp?sku=GGGKK&id=0309301913064849755057	Music box actions	A page from Klockit.com where music box actions can be purchased for $3.75 each or as low as $1.85 each when purchased in lots of 50 or more.
http://www.mfa.org/artemis/collections/mi.htm	Online collection of musical instruments from the Boston Museum of Fine Arts	A wide variety of musical instruments from many cultures can be viewed here. Images are high resolution and can be zoomed.
http://www.physicsclassroom.com/Class/sound/soundtoc.html	A tutorial on sound waves and music from *The Physics Classroom*	A fairly detailed set of tutorials geared for the high school audience. Good reinforcement with many animations and multiple choice, check-yourself problems.

http://www.ehhs.cmich.edu/~dhavlena/	Homepage of an amateur musical instrument maker	There are dozens of plans here for making simple musical instruments. There are also lots of links to other musical instrument building sites.
http://www.phys.unsw.edu.au/music/	Homepage for the University of South Wales Music Acoustics website	An excellent site for in depth study of many classes of instruments and individual instruments. There are many sound files (mp3's and wav's) that are available as demonstrations. Many, many good links as well.
http://www.Pulsethemovie.com/	Homepage for the large screen movie, *Pulse*	An excellent movie that explores the rhythmic music from many different cultures. It will change the way in which you look at and describe music. The website includes a 25-page curriculum guide probably most appropriate for middle school students.
http://www.stomponline.com/	Homepage for the stage performance, *STOMP*	The performers do a 95-minute show celebrating rhythm. As the website says, "STOMP is a movement, of bodies, objects, sounds - even abstract ideas. But what makes it so appealing is that the cast uses everyday objects, but in non-traditional ways." The website also has educational materials related to the show.
http://www.synthonia.com/artwhistling/	A history of and physical description of human whistling.	This is a mostly historical, but comprehensive coverage of Kunstpfeifen ("artwhistling"). Very little is written about one of the most available and used musical instruments – the human whistle. This is a nice overview.
http://www.tuftl.tufts.edu/mie/	Homepage for the Musical Instrument Engineering Program at Tufts University	The Musical Instrument Engineering Program at Tufts University is a both a research and teaching program. It contains useful information for those interested in developing a program for designing musical instrument and understanding musical acoustics.
http://www.windworld.com	Homepage for *Experimental Musical Instruments*	*Experimental Musical Instruments* is an organization devoted to unusual musical sound sources. There is a quarterly newsletter, an online store with many books, CD's, and supplies, lots of interesting articles and an especially impressive list of links to those active in the field of experimental musical instruments.

PHYSICS OF MUSIC RESOURCE VENDORS

VENDOR	PRODUCT AND COST	COMMENTS
Arbor Scientific PO Box 2750 Ann Arbor, MI 48106-2750 800-367-6695 http://www.arborsci.com/Products_Pages/Sound&Waves/Sound&WavesBuy1B.htm	Sound Pipes 1-11 $2.75 each 12 $1.95 each	These open-ended corrugated plastic pipes are about 30" long and 1" in diameter. A fundamental tone is produced when the sound pipe is twirled very slowly. Twirling at progressively higher speeds allows for the production of the next four harmonics (although the advertised fifth harmonic is quite hard to get). These are a lot of fun and very useful for talking about harmonics and the physics of open-end pipes.
Candy Direct, Inc. 745 Design Court, Suite 602 Chula Vista, CA 91911 619-374-2930 http://www.candydirect.com/novelty/Whistle-Pops.html?PHPSESSID=f5df897f67e7d525eadbcedfe4703e96	Whistle Pops 32 $16.45	These are candy slide whistles that really work. However, since they are candy (except for the slide), once you use them you have to either eat them or dispose of them.
Century Novelty 38239 Plymouth Road Livonia, MI 48150 800-325-6232 http://www.CenturyNovelty.com/index.aspx?ItemBasisID=444478&IndexGroupID=27&ItemID=259422 for siren whistles or http://www.CenturyNovelty.com/index.aspx?ItemBasisID=824&IndexGroupID=27&ItemID=744 for two tone whistles	2" Siren Whistles 1-11 $0.59 each 12-35 $0.40 each 36+ $0.30 each Two tone Whistles 1-11 $0.12 each 12-35 $0.10 each 36+ $.06 each	These 2" long Siren Whistles are fun to play with, can be used to discuss the production of sound, and can be used to discuss the siren effect that harmonicas and accordions use to produce tones. The Two Tone Whistles are 2.5" long and almost an inch wide. They are cheaply made, but can still be used when discussing closed pipes, beats, and dissonance.
Creative Presentation Resources, Inc. P.O. Box 180487 Casselberry, FL 32718-0487 800-308-0399 http://www.presentationresources.net/tfe_music_noisemakers.html	Groan Tubes $1.29 each Plastic Flutes $0.99 each	The Groan Tubes are 18 inches long and have a plug inside that can slide down the tube whenever it is inverted. The plug is equipped with a reed that vibrates as air moves past it as the plug slides down the tube. The tube is closed acoustically at one end and open at the other. The changing pitch produced as the plug moves down the tube makes it useful for investigating the relationship between pitch and length in a closed pipe.

Hezzie Group 3322 Sleater Kinney Road NE Olympia, WA 98506 360-459-8087 http://www.hezzie.com/cgi-bin/shop.pl/page=hezzieinstruments.html	Plastic Slide Whistles 1-23 $1.25 24 $20.00	The Slide Whistles are very cheaply made and do not work very well as slide whistles. However, if the end of the whistle is cut off and the slide removed, the whistles can be used to explore the differences between open and closed pipes (see this article).
Johnna Cossaboon Marketing Communications Manager Hohner, Inc./HSS 1000 Technology Park Drive Glen Allen, VA 23059 804-515-1900 ext. 3043	Harmonicas $2.00 each (This is an educational price that you can receive by contacting the Marketing Communications Manager directly)	These harmonicas are exceptionally well made for the cost. They can be dissected to investigate the physics of vibrating bars. Although the harmonica's reeds are driven continuously with air, as opposed to the plucked bars in a thumb piano the physics is very similar. Additionally, the process of tuning a bar can be investigated, since nearly all the reeds show evidence of scraping at the base or the tip in order to flatten or sharpen the note. Finally, musical intervals can be investigated while studying the rationale for the placement of the notes.
Klockit N2311 County Road H P.O. Box 636 Lake Geneva, WI 53147 800-556-2548 http://www.klockit.com/products/dept-166__sku-GGGKK.html	Music Box Actions 1-4 $3.75 each 5-9 $2.95 10-24 $2.55 25-49 $2.25 50+ $1.85	The Music Box Actions are an excellent resource for both introducing the physics of sound production, the use of a sound board for amplification, and for showing the relationship between the length of a vibrating bar and the corresponding frequency of vibration. There are many tunes to choose from and orders of mixed tunes can be made. Choose actions with housings that can be removed.
Talking Devices Company 37 Brown Street Weaverville, NC 28787 828-658-0660 http://www.talkietapes.com/	Talkie Tapes (sample packs) 2 $2.00 50 $20.00 75 $30.00 100 $40.00 125 $50.00 Lots of other ordering options are also available	These Talkie Tapes are 18" long red plastic strip audio recordings. They have ridges on one side and running a thumbnail over the ridges causes an audible message to be heard ("happy birthday," for example). Doing this while holding the tape against a plastic cup or piece of paper causes considerable amplification. These are very surprising for those who have never seen them before and an excellent tool for discussing sound production, sound-board amplification, and the physics of phonograph records..
The Nash Company 2179 Fourth Street Suite 2-H White Bear Lake, MN USA 55110 http://www.nashco.com/noseflutes.html	4 $3.40 8 $6.40 12 $9.25 24 $18.00 48 $35.00 100 $70.00	Nose Flutes have physics that is similar to the slide whistle. The one-piece plastic device fits over the nose and partially open mouth. Blowing gently through the nose causes air to be directed across the opening of the mouth. Changing the shape (length) of the mouth (which is the resonant cavity in this wind instrument) causes a change in pitch. This provides for a very tactile reinforcement of the relationship between the length of a closed pipe and its corresponding resonant frequency. Extensions can also be made to the physics of whistling.



"In most musical instruments the resonator is made of wood, while the actual sound generator is of animal origin. In cultures where music is still used as a magical force, the making of an instrument always involves the sacrifice of a living being. That being's soul then becomes part of the instrument and in times that come forth the "singing dead" who are ever present with us make themselves heard."

\- Dead Can Dance