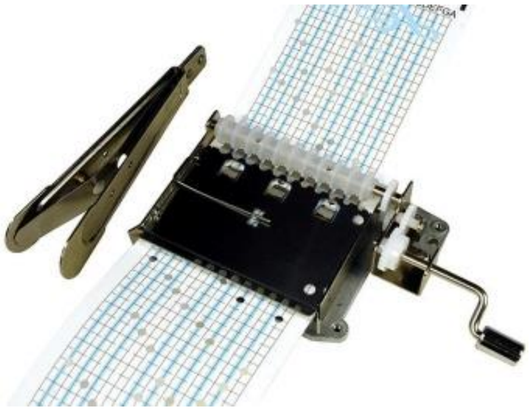
A mechanical music box

A music box (Figure 1) in itself produces hardly any sound: the vibrating tongues, like strings, are too small to set air in motion.
The solution consists of using the mechanical vibrations of the reeds, strings, etc., generated during playing, to set a soundboard in motion, and/or generate standing waves.



Figuur 1: speeldoos met ponskaart

Figuur 2: de tafel als klankbord

For example, if you place the music box on a table top (Figure 2), this top will vibrate and function as a sounding board.

The arrangement most commonly encountered is the music box mounted in a box. Of course one may wonder whether there are optimal dimensions (length L, width W and height H) for this box.

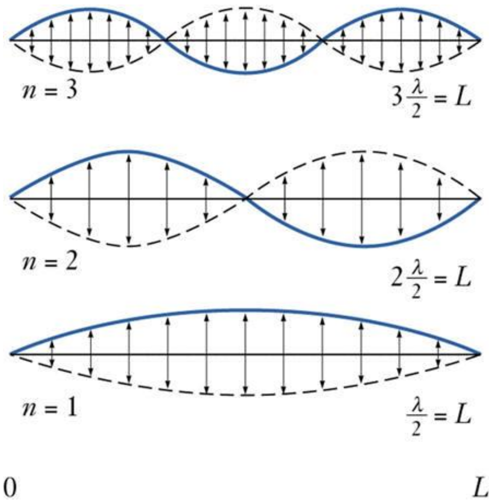
Figuur 3: muziekdoos gemonteerd in een kistje.

<u>Standing waves</u>

Through reflection and superposition, air moving between two solid walls can transform into standing waves. This phenomenon of violent vibration occurs under certain conditions, where the wavelength λ of the vibration is strictly proportional to the distance L between the two solid walls (see figure 4)

$$:n\frac{\lambda}{2} = L \qquad n = 1,2,3, \ldots$$

The standing wave with n = 1 is the most important, i.e. has the largest amplitude. We could use this for the design of the box.



Figuur 4: staande golven

The relationship between the frequency f of a sound vibration and the wavelength λ is given by:

$$\lambda = \frac{c}{f}$$

Here c is the speed of sound in air: 343 m/s. The frequency f is given in Hz (number of vibrations per second) and the wavelength λ in m.
The standing waves are then:

$$L = \frac{n}{2}\frac{c}{f}$$

or for the fundamental standing wave (n=1)

$$L = \frac{c}{2f}$$

As an example, assume that the frequency range of the music box is 1045 to 2640 Hz, which gives me the following values for the main internal dimensions

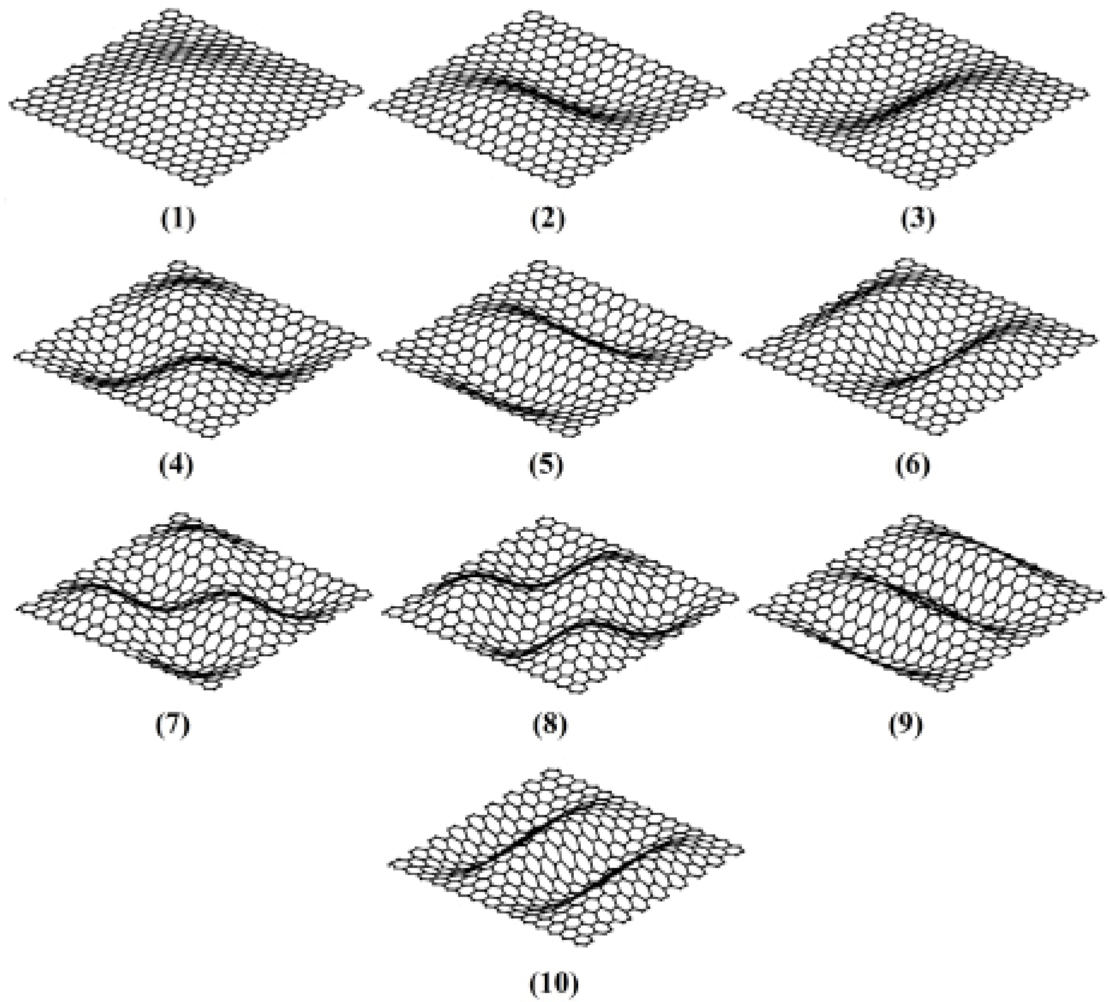
L = 343/2090 = 0.164 m or 16.4 cm
B = 343/3322 = 0.103 m or 10 cm
H = 343/5280 = 0.065 m or 6.5 cm

The main dimensions must of course always allow that the music box can be built into the box.

Sound board

The vibrating tongues and the standing waves in the box will ideally vibrate the walls of the box and cause the music box to produce sound. Since the main dimensions are given, the choice that remains is: the type of wood and the thickness of the walls.

Wood is an orthotropic material, i.e. the mechanical properties strongly depend on the direction: a large elastic modulus EL in the longitudinal direction of the wood and an approximately 10 times smaller elastic modulus ER in the radial direction. What we need to investigate are the bending modes and associated frequencies of a rectangular sandwiched plate with thickness t, specific mass ρ and elastic moduli EL and ER .



Figuur 5: trillingsmoden van een vierkante plaat

A rectangular plate (LxW) that is clamped on the four sides can vibrate in (in principle) an infinite number of ways. Figure 5 shows the first ten vibration modes for a square plate (the ones for a rectangular plate are similar). Each mode has a frequency that increases with the complexity of the mode.
Without going into the derivation (which is approximate in itself), we can say that for a rectangular soundboard (length L, width B), with the fiber direction of the wood parallel to the longest side L, the vibration frequency f1 of the first mode is given by

$$f_1 = \frac{23{,}2}{2\pi}\frac{t}{L^2}\sqrt{\frac{E_L}{12\rho(1-\nu^2)}}$$

Here t is the thickness of the sheet, L is the long side, EL is the modulus of elasticity in the longitudinal direction, ρ is the specific mass, and v is the transverse contraction.
In Table 1 the typical values of these quantities are given for spruce wood.

For L = 0.16 m and a frequency f1 =1000 Hz, a thickness t = 4 mm follows. At f1 = 2000 Hz comes a thickness of 8 mm

	picea abies (vurenhout)	
soortelijke massa ρ	400 kg/m3	
elasticiteitsmodulus E_L	10 GPa	
elasticiteitsmodulus E_R	1 GPa	
dwarscontractie	0,5	

Table 1 Physical properties of spruce wood

If the grain direction of the wood is taken parallel to the shorter side, the following applies:

$$f_1 = \frac{36}{2\pi}\frac{t}{B^2}\sqrt{\frac{E_L}{12\rho(1-\nu^2)}}$$

and we find, with B=0.10 m, for the thickness t at 1000 and 2000 Hz respectively 1 mm and 2 mm.

<u>Conclusion</u>

The dimensions of the box (L, W, H) can be chosen in function of standing waves that have a frequency in the range of the music box.
The lid (and bottom) with the dimensions LxW can be designed as a sounding board. With spruce wood and the grain direction parallel to the long side, one can choose a thickness of about 4 mm. But because the above analysis is approximate, and also because the chosen wood can have other properties, it is good to experiment with the thickness.

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